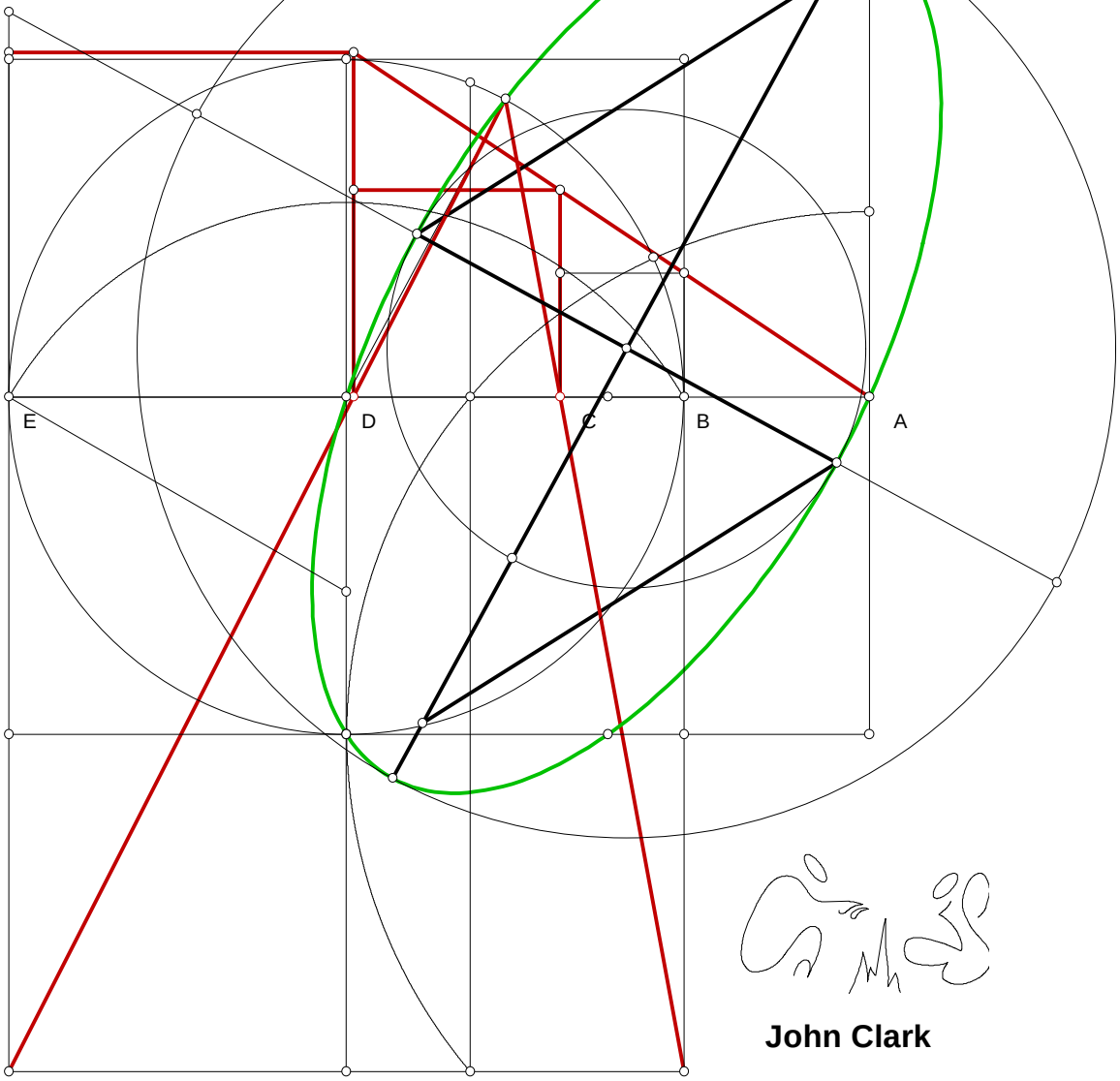
The Delian Quest

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$



John Clark

The Delian Quest

Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. The smattering of English is of no great consequence. The figure is Geometric Grammar, the equations are Algebraic Grammar.

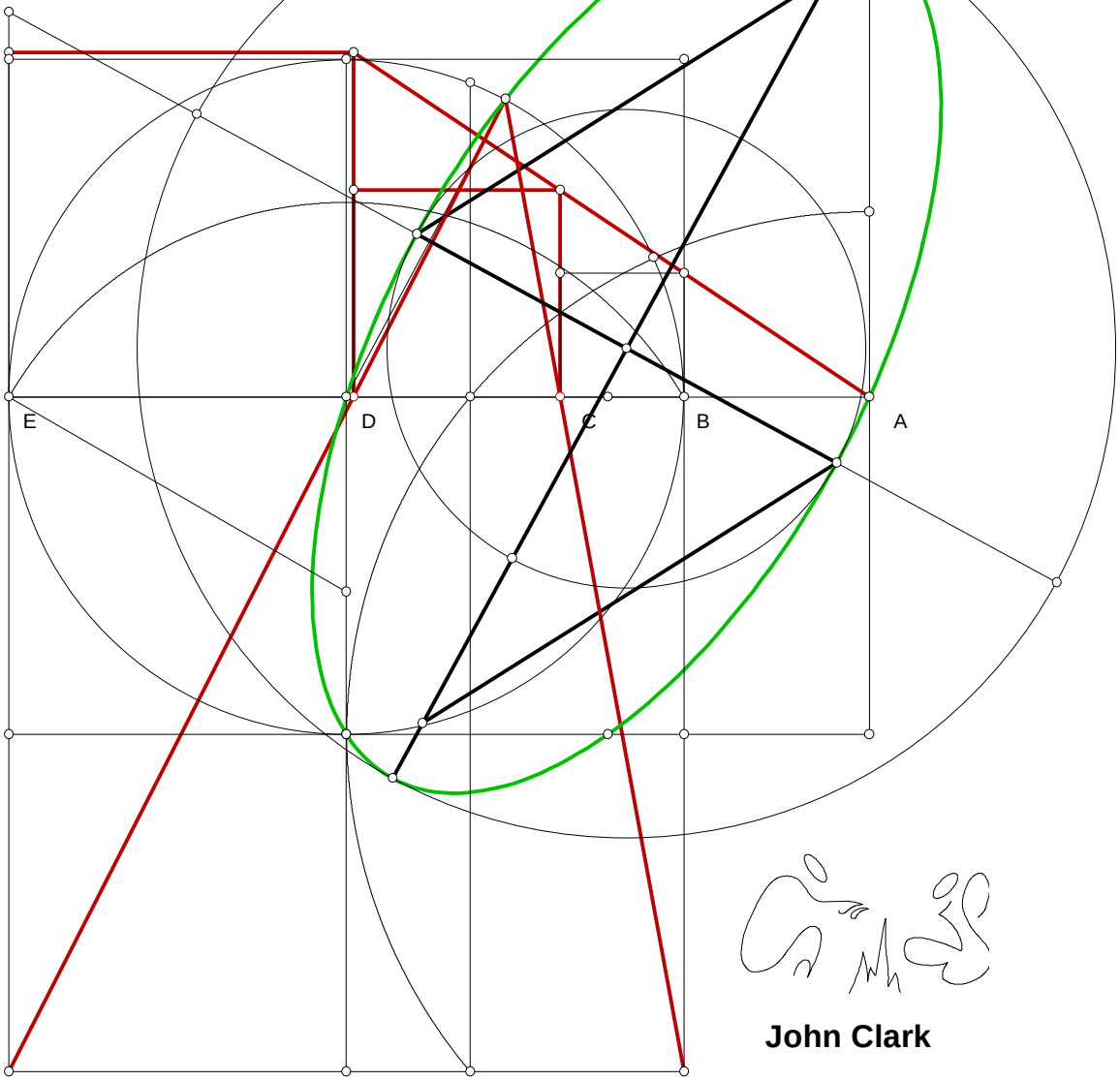
Place Introduction to logic here.

The Delian Quest

1989

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

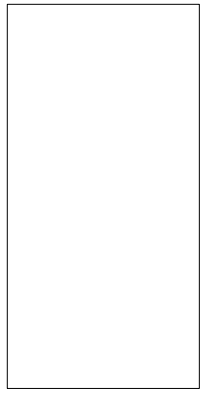


Plate 1

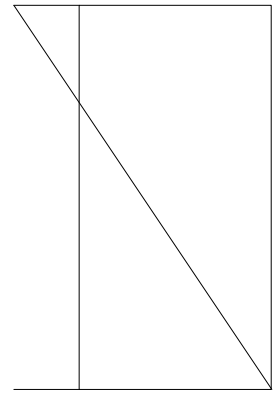


Plate 2

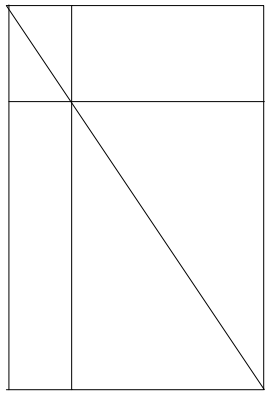


Plate 3

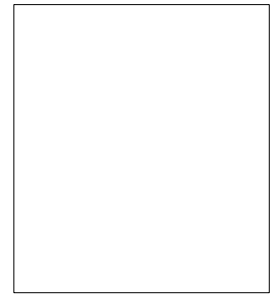


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

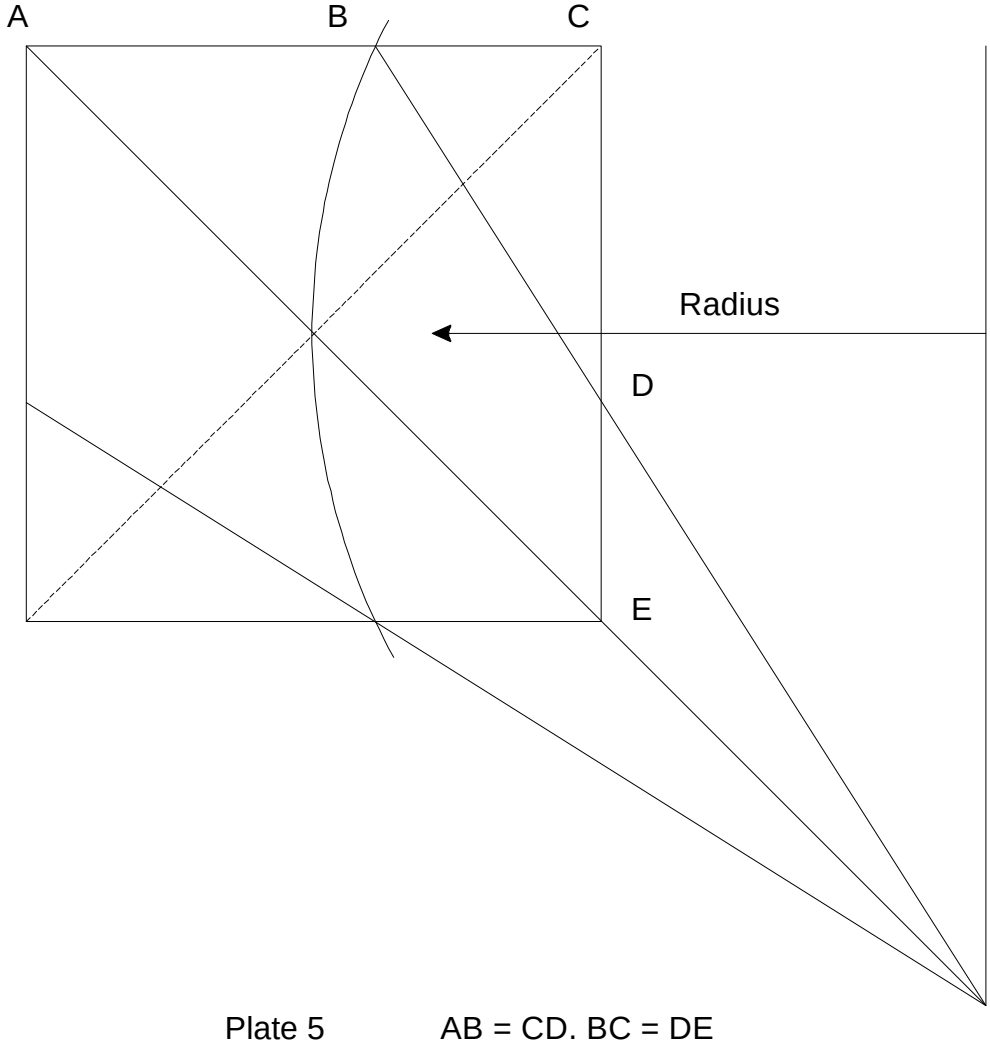
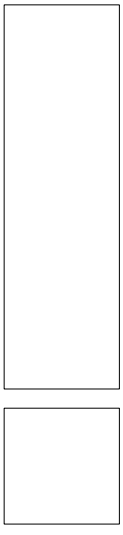
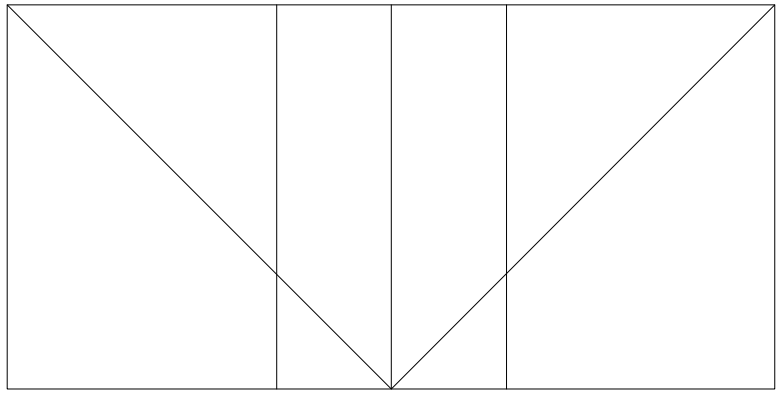


Plate 5 AB = CD. BC = DE

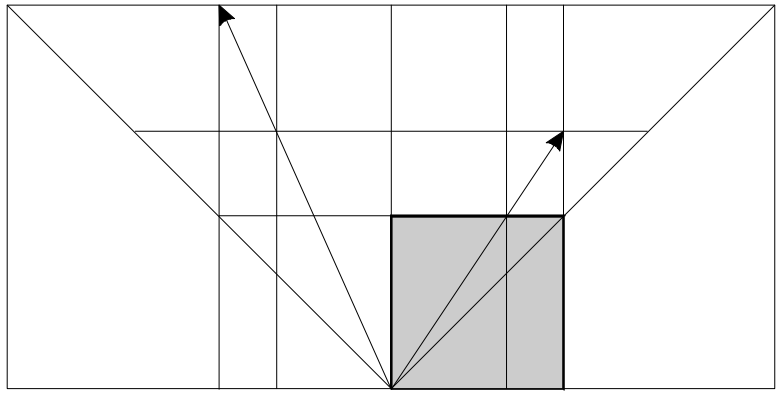
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



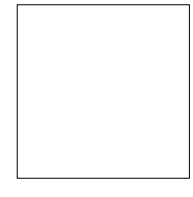
P.6



P.7

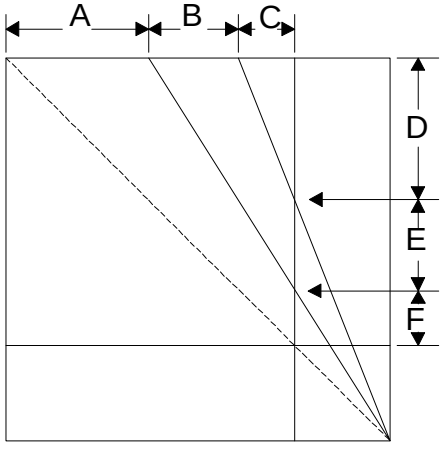


P.8

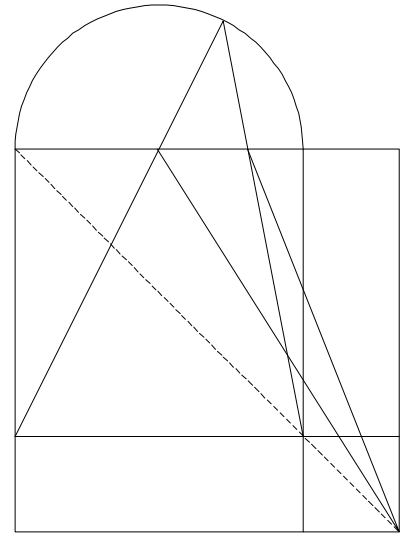


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

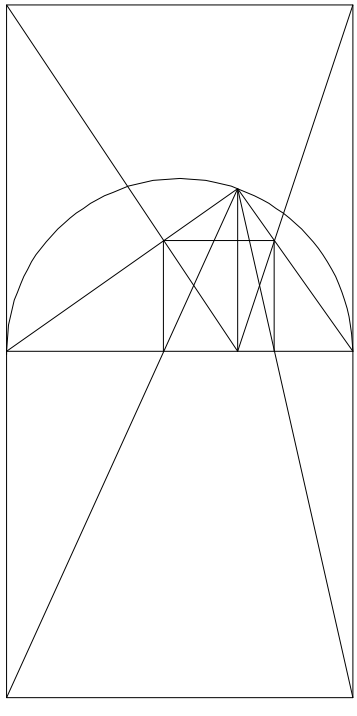


P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

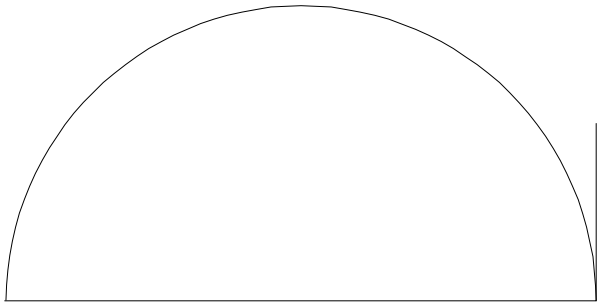


Plate 13

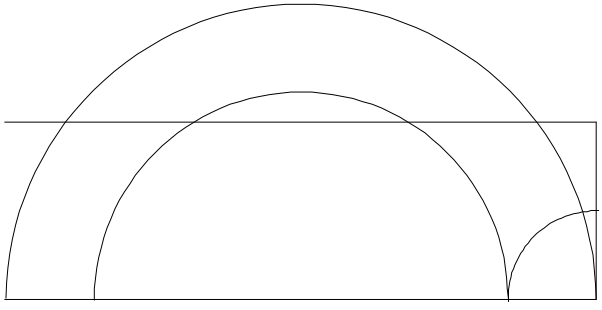


Plate 14

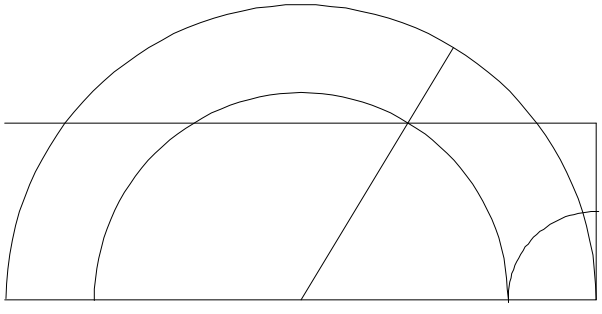


Plate 15

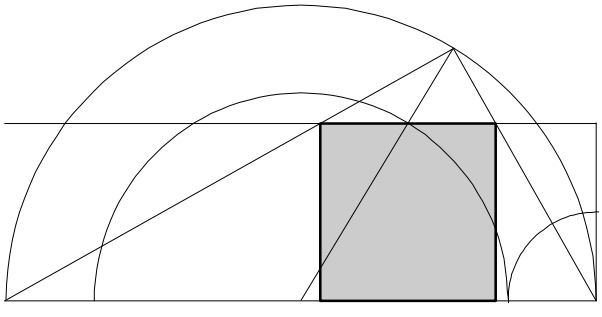
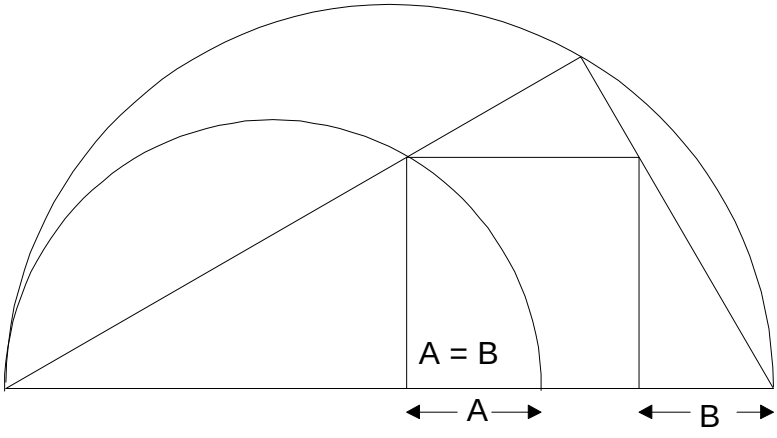
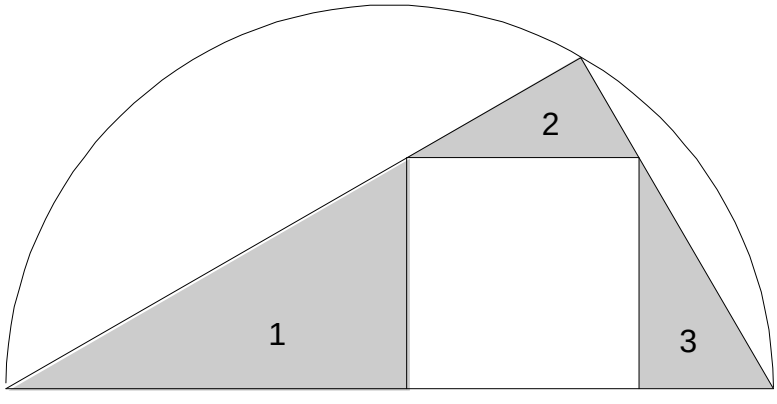


Plate 16

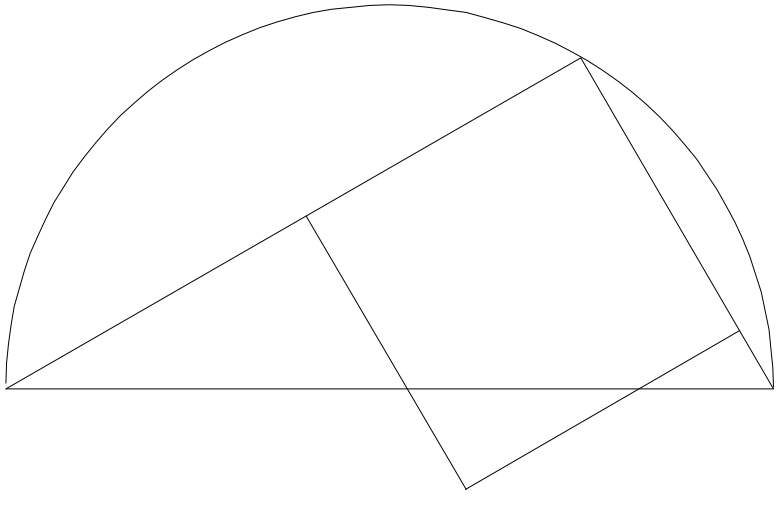
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.



In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"



P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

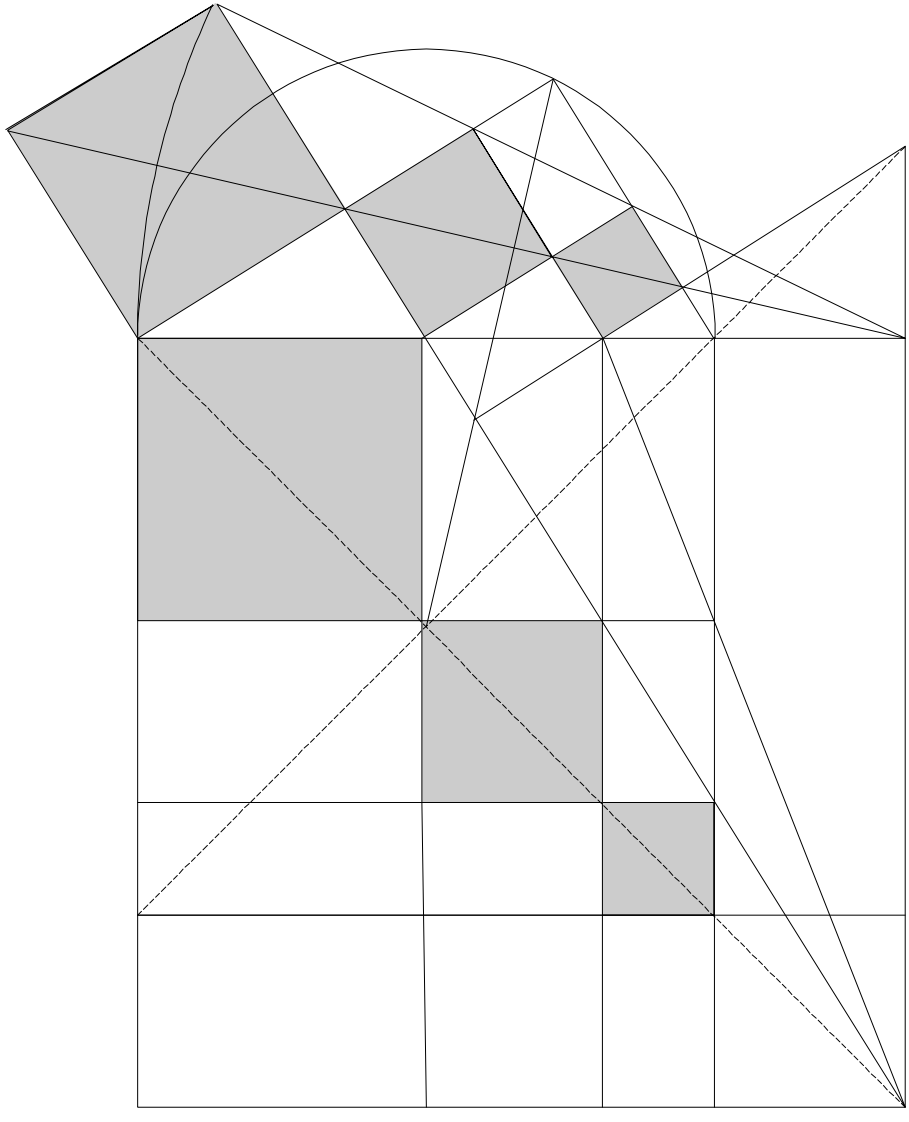
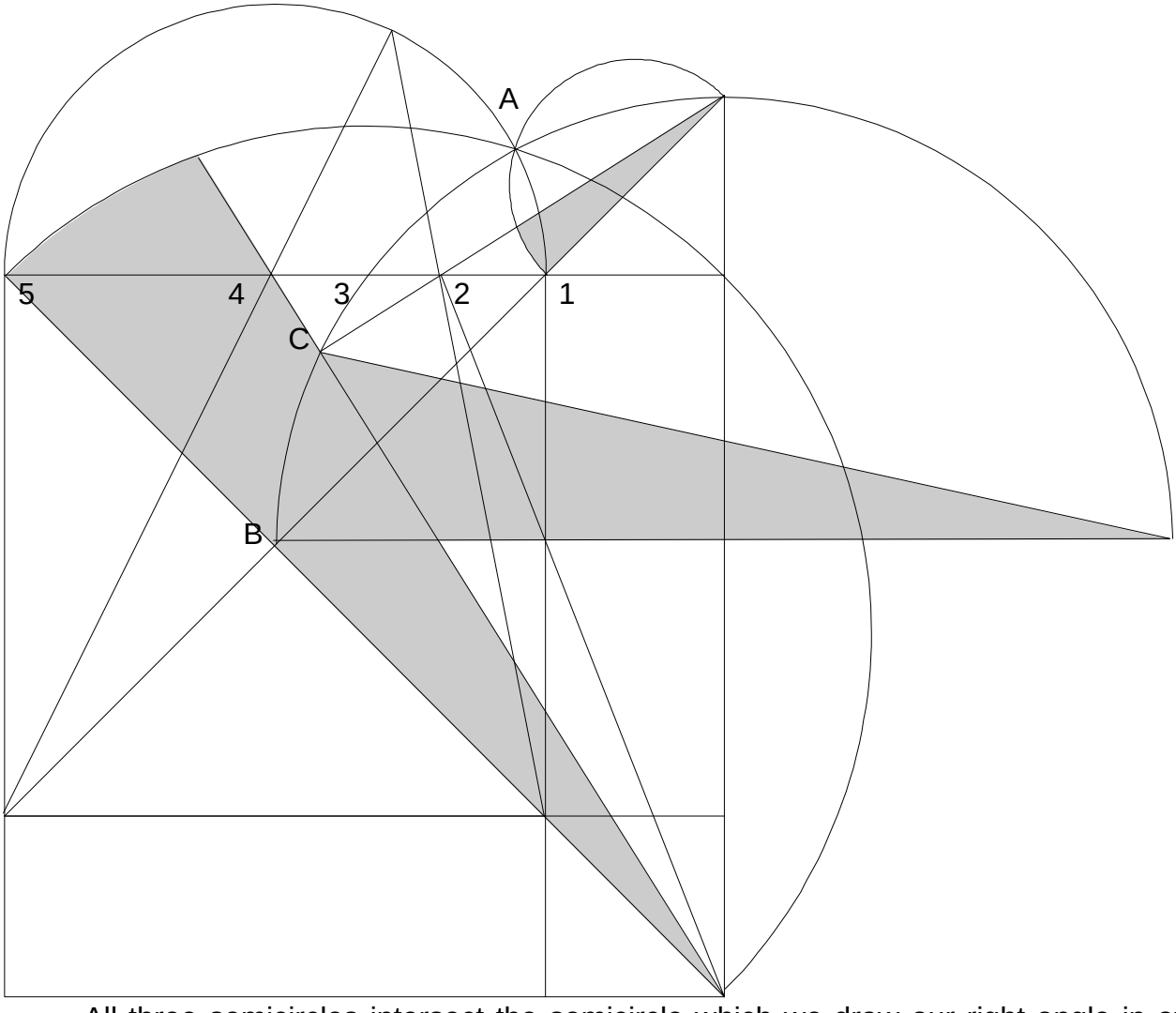


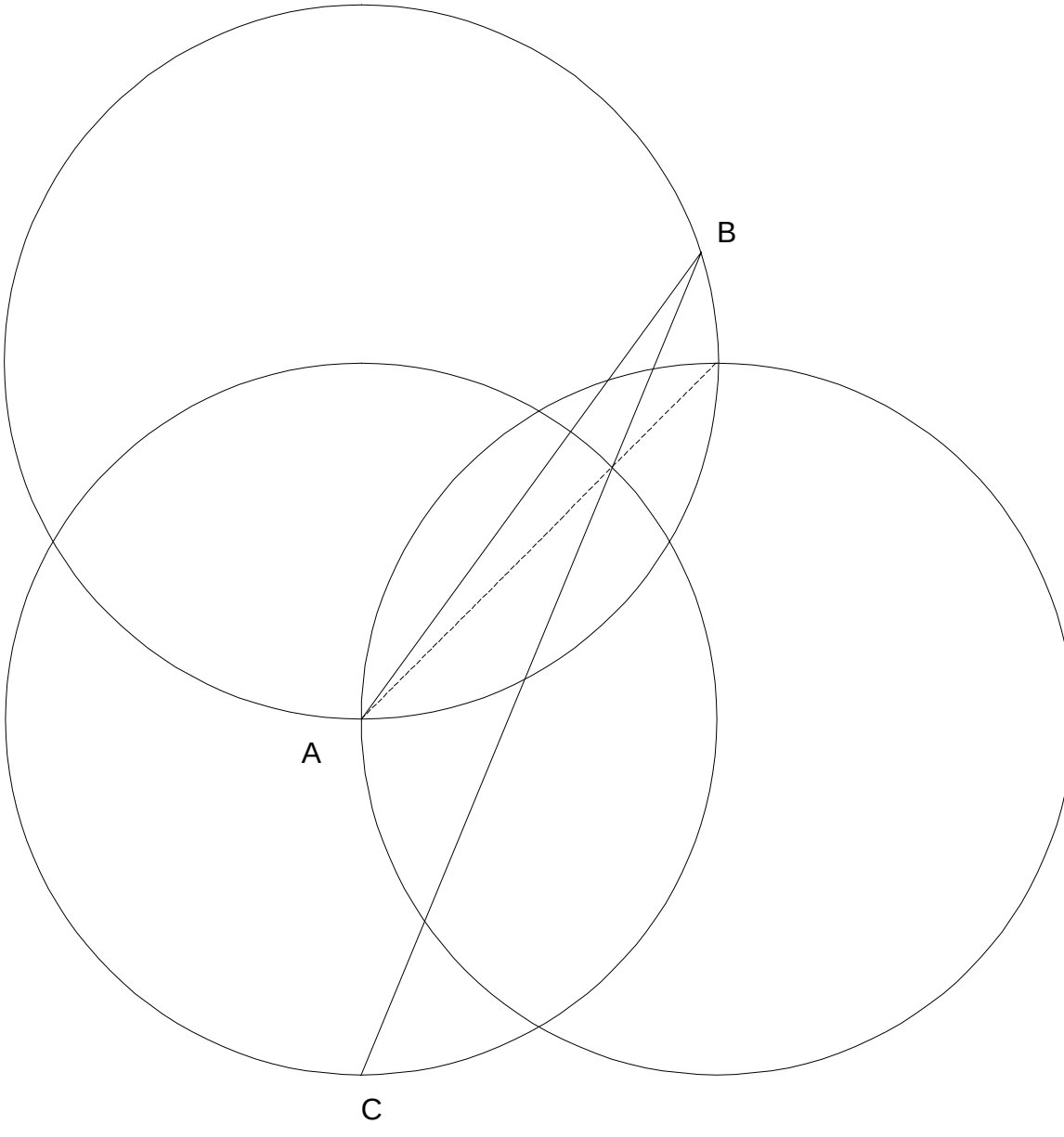
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.



All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.



How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

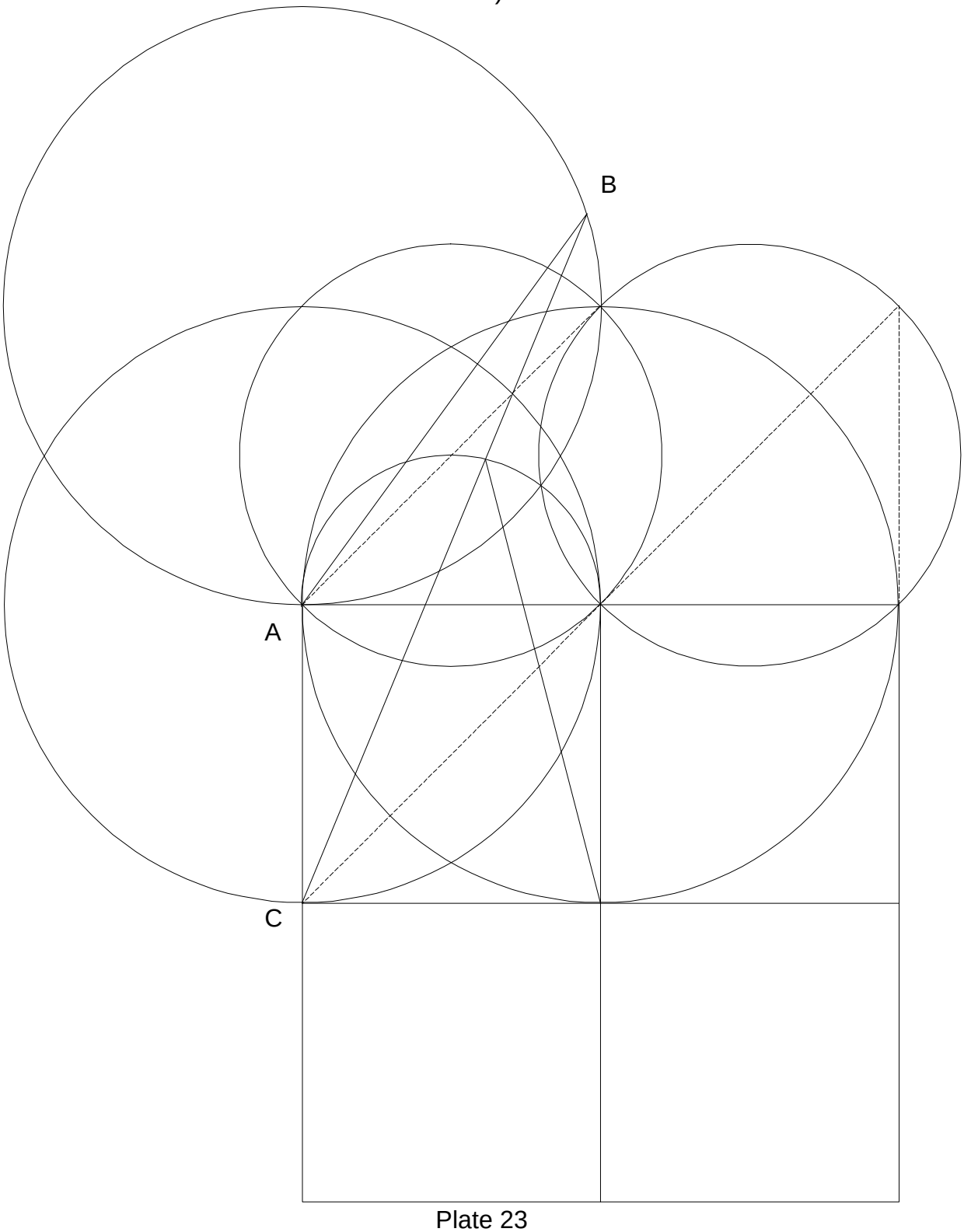
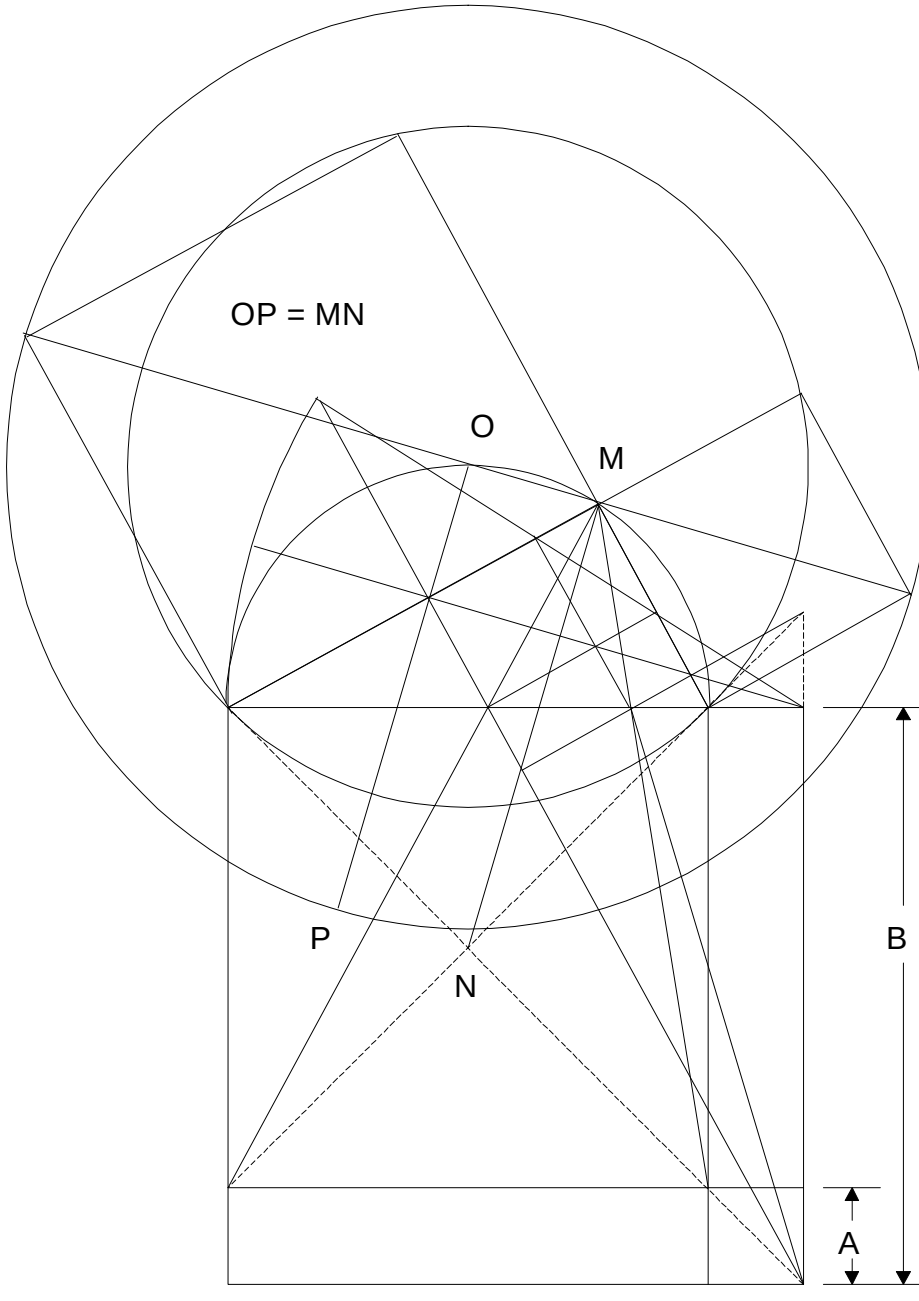


Plate 23

On Plate 24 the radius for the circle OP is given by MN.



One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of B^2A (if you have missed it, the figure gives both roots, A^2B and B^2A) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

GEOMETRIAE DEDICATA

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Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

Location:
201 Charles **Street**
Providence, **RI**
02904

December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

Serving the mathematical community for over 100 years

American Mathematical Society

Department of Mathematics
Roger E. Howe
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

société mathématique defrance

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

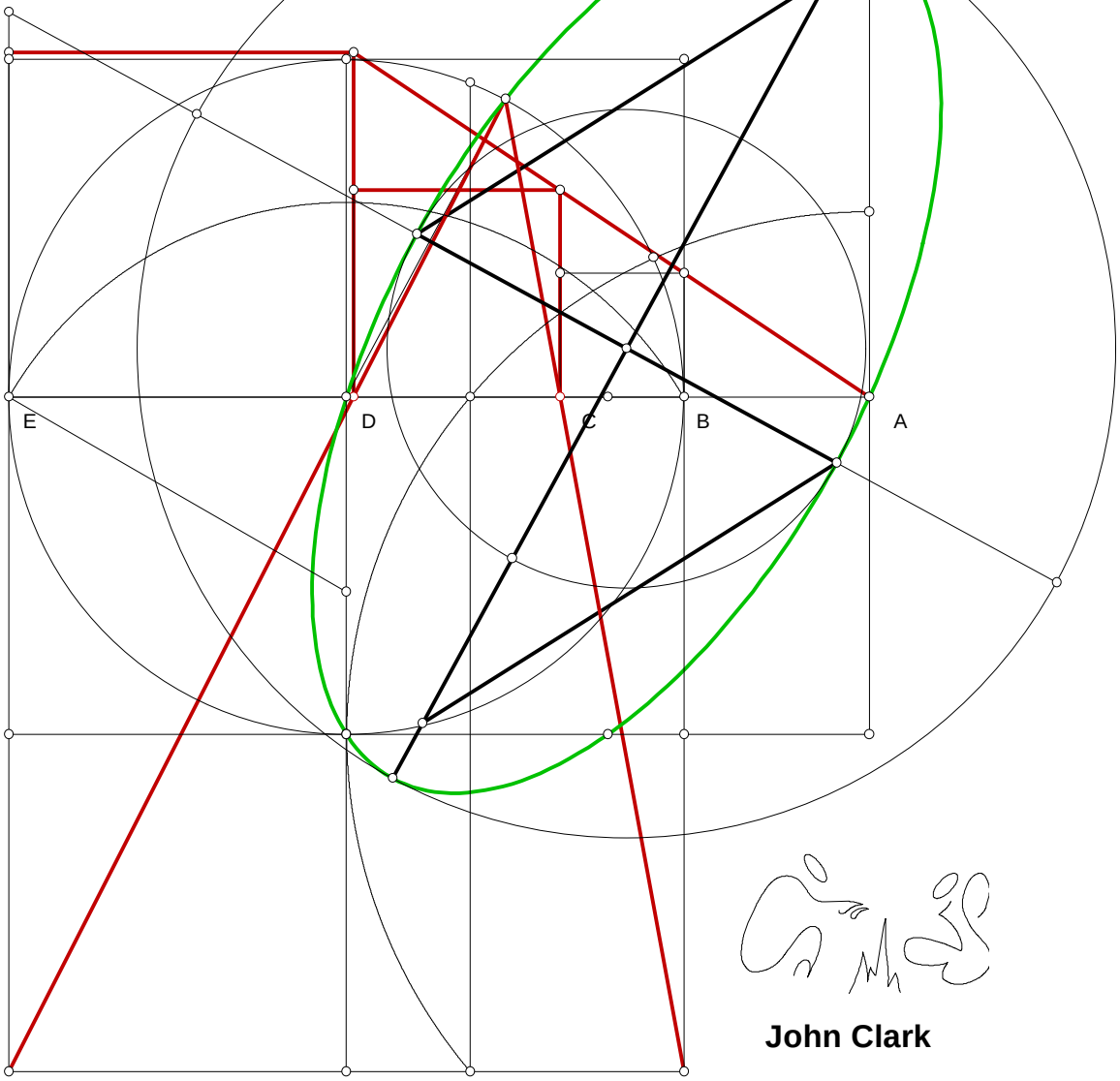
P.J. : Manuscrit

The Delian Quest

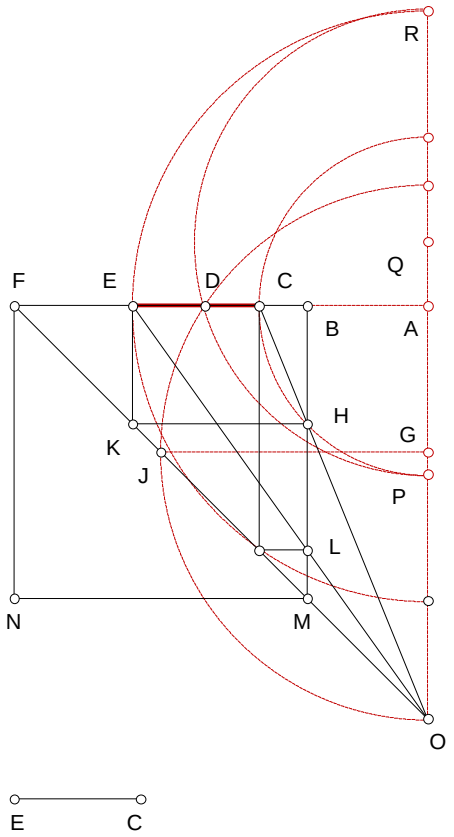
1992

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark



A Triplicate Ratio 06/20/92

Given some point O place CE on BF such that O is the point of similarity.

$N := 1.52 \quad N_2 := .89 \quad N_3 := .66$

$BF := N \quad MO := N_2 \quad CE := N_3$

$FM := \sqrt{2 \cdot BF^2} \quad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF} \quad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \quad QR := DQ \quad QP := DQ$

$AP := QP - AQ \quad AR := QR + AQ \quad AC := AP$

$AE := AR \quad AO := AF \quad BC := AC - AB$

$EF := AF - AE \quad EK := EF \quad BH := EK$

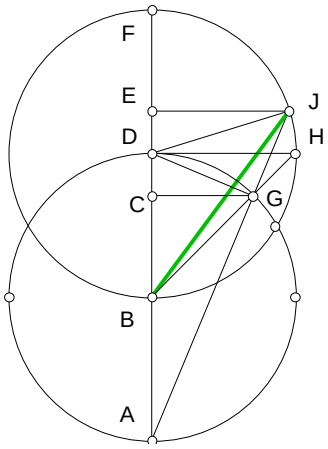
$\frac{BC}{BH} - \frac{AC}{AO} = 0$

The last ratio can be tediously proved by reducing each term to the givens.

Edit 062800

08/12/92 Rusty Cube of a Sphere

Given AB, how close is BJ to the cube root of AB taken as a sphere?



$N := 9$ $\quad CUBE_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

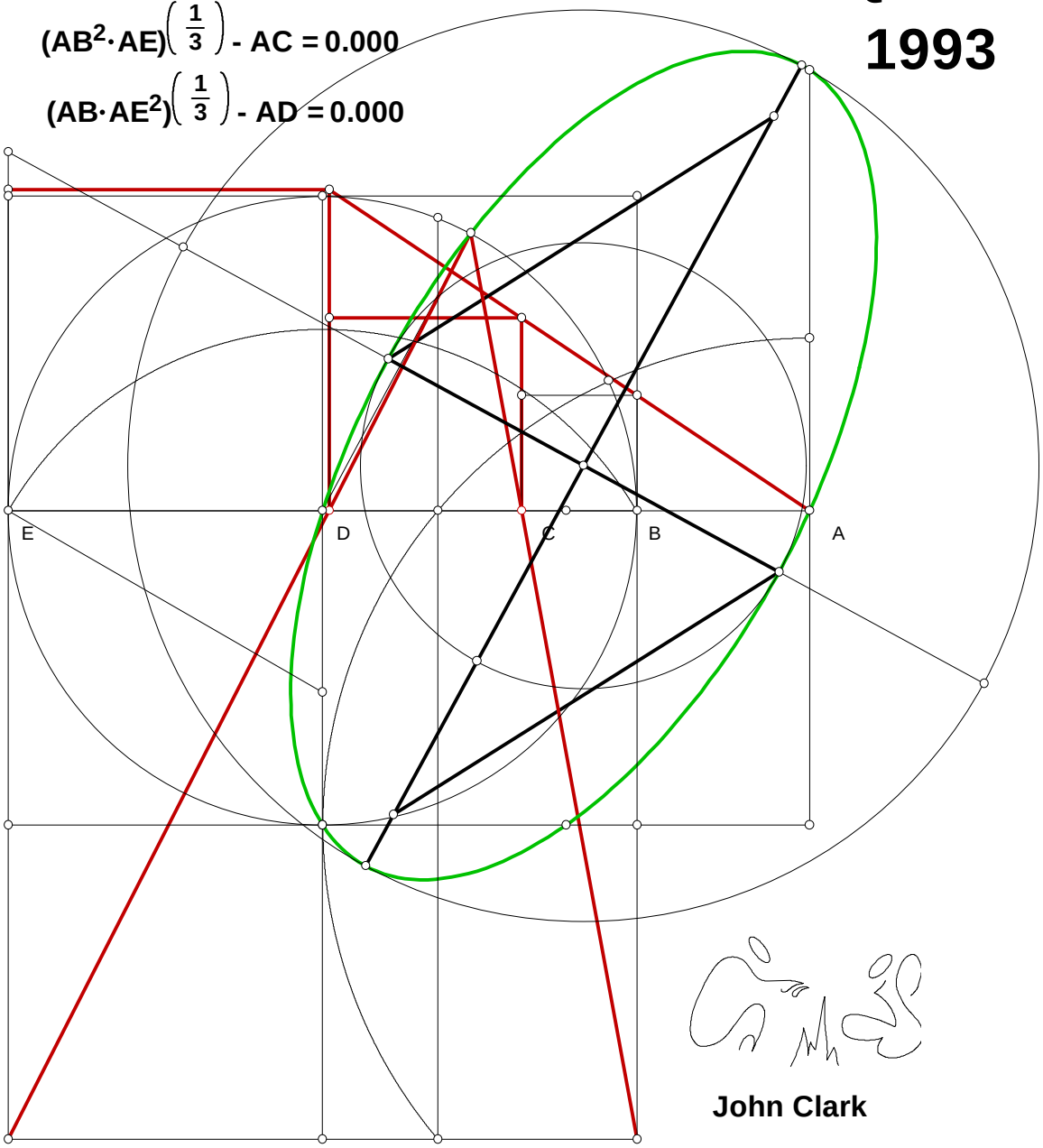
$\frac{CUBE_ROOT}{BJ} = 0.99906$

The Delian Quest

1993

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

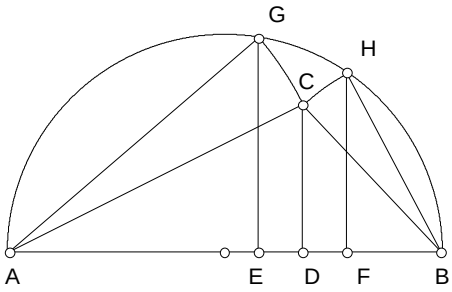
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

01/08/93 Pythagoras Revisited

Given AB, BC, AC, what is CD and CJ?



$S_1 := 2.24 \quad S_2 := 1.046 \quad S_3 := 1.7$

$S_1 := S_1 \quad S_2 := S_2 \quad S_3 := S_3$

$AB := S_1 \quad BC := S_2 \quad AC := S_3$

$AG := AC \quad BH := BC \quad AE := \frac{AG^2}{AB}$

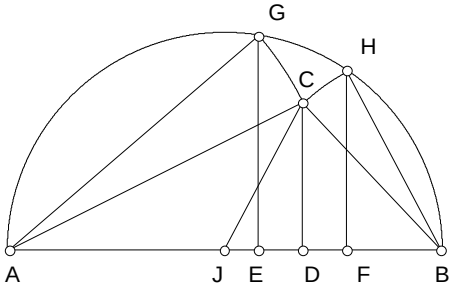
$BF := \frac{BH^2}{AB} \quad EF := AB - (AE + BF)$

$DE := \frac{EF}{2} \quad AD := AE + DE \quad AD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$BD := BF + DE \quad BD := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CD := \sqrt{AC^2 - AD^2}$

$CD := \frac{\sqrt{(-S_1 + S_2 - S_3) \cdot (S_1 + S_2 + S_3) \cdot (S_1 - S_2 - S_3) \cdot (S_1 + S_2 - S_3)}}{2 \cdot S_1}$



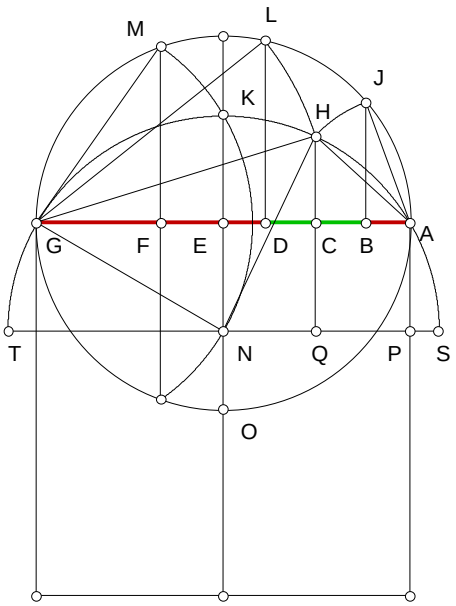
$AJ := \frac{AB}{2} \quad JD := AD - AJ \quad CJ := \sqrt{JD^2 + CD^2}$

$CJ := \frac{1}{2} \cdot \sqrt{2 \cdot S_3^2 - S_1^2 + 2 \cdot S_2^2}$

$(AC^2 + BC^2) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).

06/03/93.MCD Exploring The Curve CJ



Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?

$N := 2 \qquad N_2 := 4$

$AG := N \qquad AC := \frac{AG}{N_2} \qquad GF := \frac{AG}{3} \qquad AE := \frac{AG}{2}$

$EG := AE \qquad AF := AG - GF \qquad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \qquad GN := GM \qquad EN := \sqrt{GN^2 - EG^2}$

$NH := GM \qquad NS := GM \qquad PN := AE \qquad PS := NS - PN$

$ST := 2 \cdot GM \qquad SQ := AC + PS \qquad QT := ST - SQ$

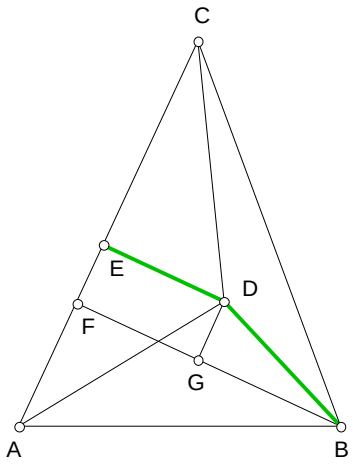
$QH := \sqrt{SQ \cdot QT} \qquad CQ := EN \qquad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \qquad CG := AG - AC \qquad GH := \sqrt{CG^2 + CH^2} \qquad AJ := AH \qquad AB := \frac{AJ^2}{AG}$

$GL := GH \qquad DG := \frac{GL^2}{AG} \qquad BD := AG - (AB + DG) \qquad \sqrt{AB \cdot DG} - BD = 0$

$$\left(\frac{-N}{3} + \frac{N}{3} \cdot \frac{\sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2} \right) - BD = 0$$

06/07/93 For All Triangles Find BD



Given AB, BC, AC, CD, AD, find BD

To simplify use line names found in 010893

$S_1 := 5.55$ $S_2 := 4.61$ $S_3 := 1.5$

$S_5 := 4.5$ $S_6 := 5.48$

$AB := S_5$ $BC := S_6$ $AC := S_1$ $CD := S_3$ $AD := S_2$

$$DE := \frac{\sqrt{\left(-S_1 + S_2 - S_3\right)\cdot\left(S_1 + S_2 + S_3\right)\cdot\left(S_1 - S_2 - S_3\right)\cdot\left(S_1 + S_2 - S_3\right)}}{2\cdot S_1}$$

$$BF := \frac{\sqrt{\left(-S_1 + S_5 - S_6\right)\cdot\left(S_1 + S_5 + S_6\right)\cdot\left(S_1 - S_5 - S_6\right)\cdot\left(S_1 + S_5 - S_6\right)}}{2\cdot S_1}$$

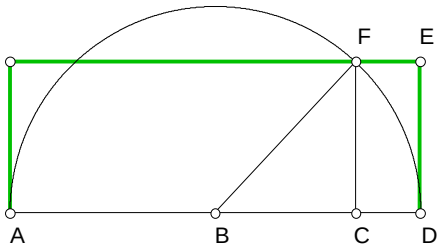
$CE := \sqrt{CD^2 - DE^2}$ $CF := \sqrt{BC^2 - BF^2}$ $EF := CF - CE$ $GF := DE$

$BG := BF - GF$ $DG := EF$ $BD := \sqrt{DG^2 + BG^2}$ $BD = 3.983$

OR

$BG_2 := BF + GF$ $DG_2 := EF$ $BD_2 := \sqrt{DG^2 + BG_2^2}$ $BD_2 = 5.757$

06/09/93 Rectangular Roots



Given AD and DE divide AD into the rectangular roots of DE.

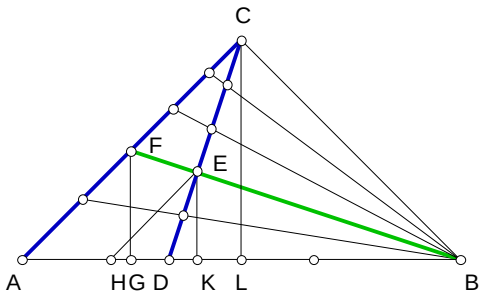
$N := 5 \qquad N_2 := 2$

$AD := N \quad DE := N_2 \quad CF := DE \quad BF := \frac{AD}{2}$

$AB := BF \quad BC := \sqrt{BF^2 - CF^2} \quad AC := AB + BC \quad CD := AD - AC \quad \sqrt{CD \cdot AC} - DE = 0$

$AC := \frac{1}{2} \cdot N + \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2} \qquad CD := \frac{1}{2} \cdot N - \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2}$

06/21/93 Pyramid of Ratios I



Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?

$N1 := 3 \qquad N2 := 5 \qquad \delta := 1 .. N2$

$AB := 1 \qquad AD := \frac{AB}{N1} \qquad AL := \frac{AB}{2}$

$DL := AL - AD \qquad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \qquad CD := \sqrt{DL^2 + CL^2}$

δ
1
2
3
4
5

$DE_\delta := \frac{CD \cdot \delta}{N2} \qquad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \qquad AK_\delta := AD + DK_\delta \qquad BK_\delta := AB - AK_\delta \qquad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \qquad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \qquad BH_\delta := BK_\delta + HK_\delta \qquad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \qquad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \qquad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

3.75
5
7.5
15
0

$\text{if}\left(N2 - \delta, \frac{N1 \cdot N2}{N2 - \delta}, 0\right)$

3.75
5
7.5
15
0

$\frac{AC}{AF_\delta}$

3.667
2
1.444
1.167
1

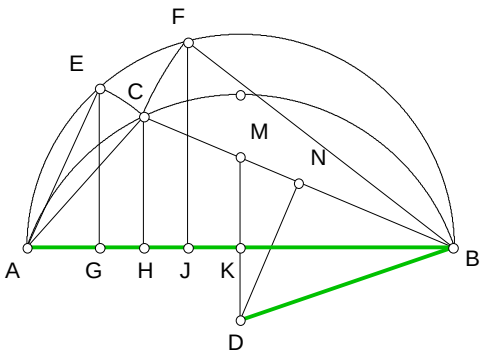
$\frac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta}$

3.667
2
1.444
1.167
1

06/27/93 Describe A Circle About a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $\quad NOT(X) := X = 0$ $\quad \delta := 0 .. 2$



$BK := \frac{AB}{2}$ $\quad AE := AC$ $\quad BF := BC$

$AG := \frac{AE^2}{AB}$ $\quad BJ := \frac{BF^2}{AB}$ $\quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $\quad BH := BJ + HJ$ $\quad CH := \sqrt{BC^2 - BH^2}$

$BN := \frac{BC}{2}$ $\quad BM := \frac{BC \cdot BK}{BH}$ $\quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $\quad BD := \sqrt{BN^2 + DN^2}$

$radius := if(\Delta, BD, 0)$ $\quad imaginary_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary_radius = 0$

$\Delta = 1$

$S_1 := \begin{bmatrix} AB \\ AC \\ BC \end{bmatrix}$ $\quad S_2 := \begin{bmatrix} AC \\ BC \\ AB \end{bmatrix}$ $\quad S_3 := \begin{bmatrix} BC \\ AB \\ AC \end{bmatrix}$

$AB \equiv 3$ $\quad AC \equiv 4$ $\quad BC \equiv 6$

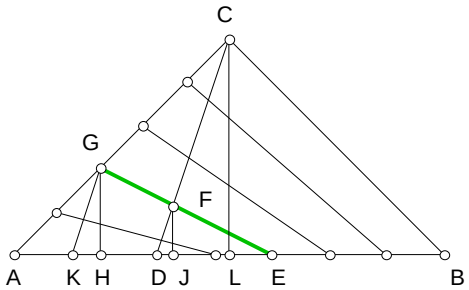
The name of the Radius as a proportion of the given names

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = [\ 3.375 \quad 3.375 \quad 3.375\ ]$

07/15/93 Pyramid of Ratios II

AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?



$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

1.5
2.333
4
9
0

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

1.5
2.333
4
9
0

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

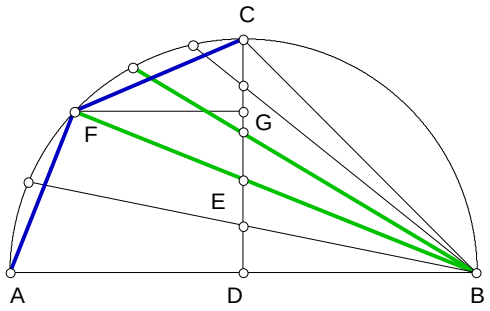
15
4.375
2.222
1.406
1

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

15
4.375
2.222
1.406
1

07/25/93 Pyramid of Ratios III

Dividing DC by an number provides wht in terms of BE/BF and AF/CF?



$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

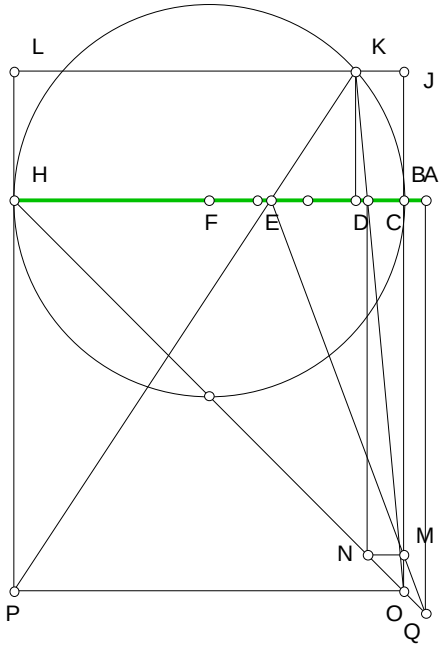
$DG := DE + EG \qquad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

$\frac{DE_\delta}{DG_\delta}$	$\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$	$\frac{BE_\delta}{BF_\delta}$	$\frac{BF_\delta}{BE_\delta}$	$\frac{2}{\left(\frac{\delta}{N}\right)^2 + 1}$
0.514	0.514	0.514	1.946	1.946
0.556	0.556	0.556	1.8	1.8
0.625	0.625	0.625	1.6	1.6
0.722	0.722	0.722	1.385	1.385
0.847	0.847	0.847	1.18	1.18
1	1	1	1	1

$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$	$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$
0.283	0.283
0.707	0.707
1.414	1.414
2.828	2.828
7.071	7.071
0	0

11/06/93 Gruntwork I on the Delian Solution



Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?

$N := 4$ $BH := 1$

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $BJ := DK$ $BO := BH$

$JO := BJ + BO$ $JK := BD$ $CD := \frac{JK \cdot DK}{JO}$

$KL := DH$ $LP := JO$ $DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD$ $CE := CD + DE$ $MN := BC$

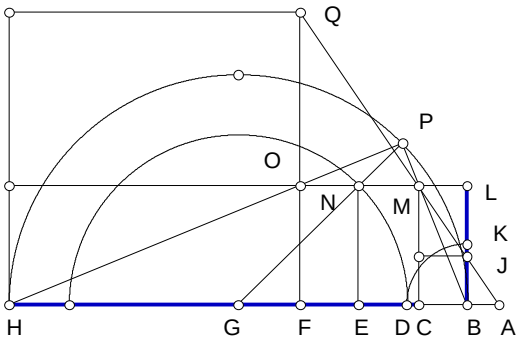
$GH := MN$ $CH := BH - BC$ $HN := \sqrt{2 \cdot CH^2}$ $GM := HN$ $EH := CH - CE$ $EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG}$ $HO := \sqrt{2 \cdot BH^2}$ $OQ := HQ - HO$ $OR := \sqrt{\frac{OQ^2}{2}}$ $AB := OR$ $AC := AB + BC$

$AE := AB + BD + DE$ $AH := AB + BH$ $\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$

11/09/93 Solve For Cube Root Placement

With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



$N := 2 \quad BH := N - 1$

$BG := \frac{BH}{2} \quad CF := N^{\frac{2}{3}} - N^{\frac{1}{3}}$

$BL := CF \quad GP := BG \quad BK := \frac{BL}{2}$

$BD := BK \quad NP := BD \quad GN := GP - NP$

$EN := BL \quad GE := \sqrt{GN^2 - EN^2}$

$CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD$

$FH := GH + GE - EF \quad FQ := FH \quad FO := BL \quad OQ := FQ - FO \quad MO := CF \quad AF := \frac{MO \cdot FQ}{OQ}$

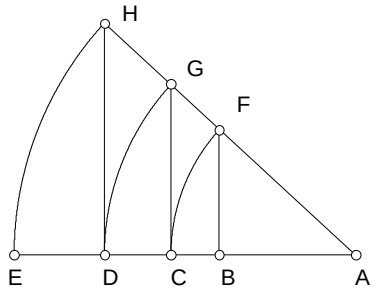
$AC := AF - CF \quad AH := AF + FH \quad AB := AH - BH$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 2$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

11/10/93 Gruntwork II on the Delian Solution

Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?



$N := 1.1 \qquad AE := 10$

$DE := \frac{AE}{N} \quad AD := AE - DE \quad AH := AE$

$AG := AD \quad AC := \frac{AD \cdot AD}{AE} \quad AF := AC$

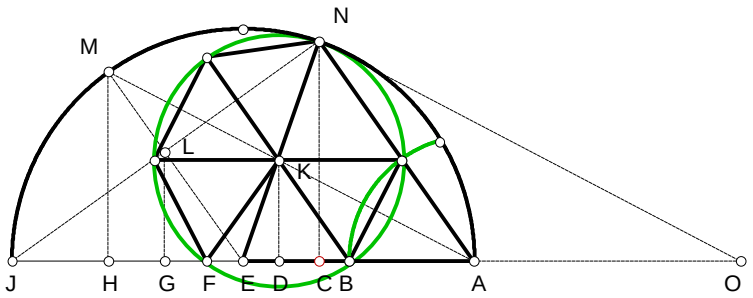
$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 1331 \quad \frac{AD}{AB} = 121 \quad \frac{AC}{AB} = 11$

11/11/93 The Archamedian Paper Trisector

If one accepts the facts of the original figure, one only need prove that BK = AB.
If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?



$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN \cdot JL}{JN}$ $GJ := \frac{CJ \cdot JL}{JN}$

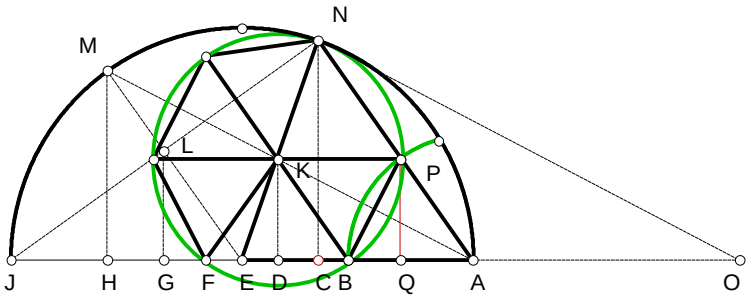
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

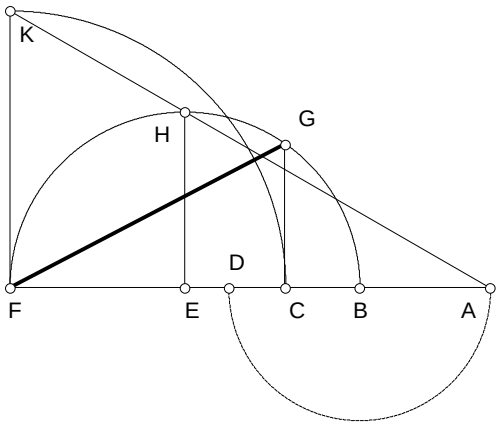
$AB - BK = 0$ $AB = 0.25$

If PK is parallel to AJ, then . . . .



$AN := \sqrt{AC^2 + CN^2}$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

11/12/93 To Square A Circle Off The Base Of A Right Triangle.



Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?

Using the approximation,π = 22/7, square the circle off the base of a right triangle.

$BF := 1$ $BE := \frac{BF}{2}$ $EF := BE$ $EH := BE$ $BD := \frac{3}{4} \cdot BE$ $AB := BD$

$AE := AB + BE$ $AF := AB + BF$ $FK := \frac{EH \cdot AF}{AE}$ $CF := FK$ $BC := BF - CF$

$CG := \sqrt{BC \cdot CF}$ $FG := \sqrt{CF^2 + CG^2}$ $\pi_A := \frac{FG^2}{BE^2}$

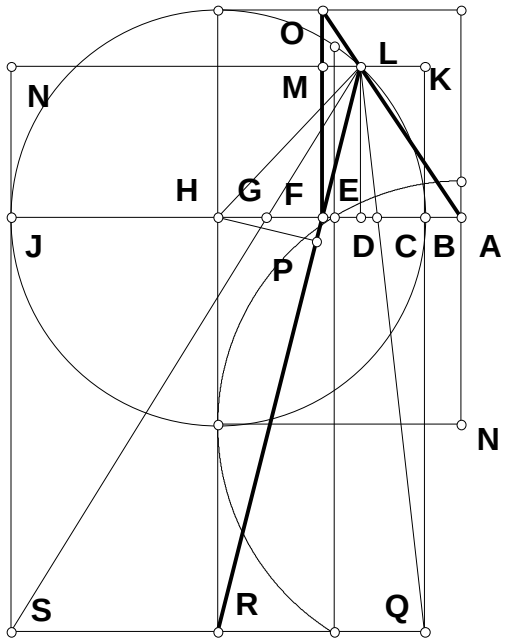
$\pi = 3.14159265359$

$\pi_A = 3.142857142857$

$\frac{\pi}{\pi_A} = 0.999597662505843$

11/18/93 Exploring Cube Roots Plate A

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.



$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad MO := FO - DL \quad LM := DF$

$AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF \quad BQ := BJ$

$BK := DL \quad BD := BH - (FH + DF)$

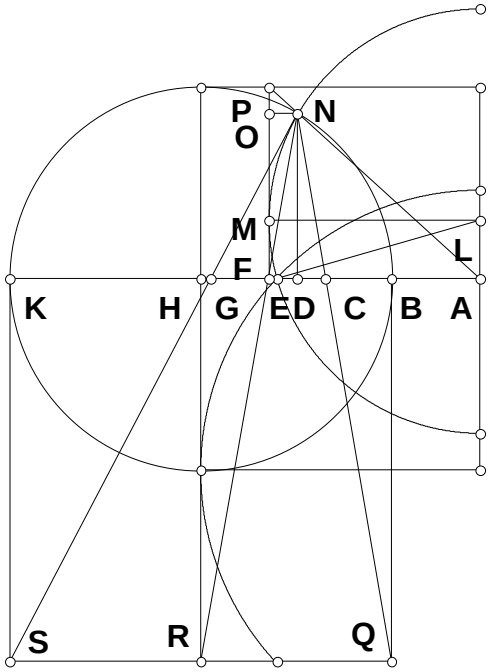
$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad HJ := BH \quad DJ := HJ + FH + DF \quad LN := DJ$

$JS := BJ \quad JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC$

$AG := AB + BG \quad AJ := AB + BJ \quad \left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

11/18/93 Exploring Cube Roots Plate B

If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.



$N := 1.2$ $BK := 1$

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

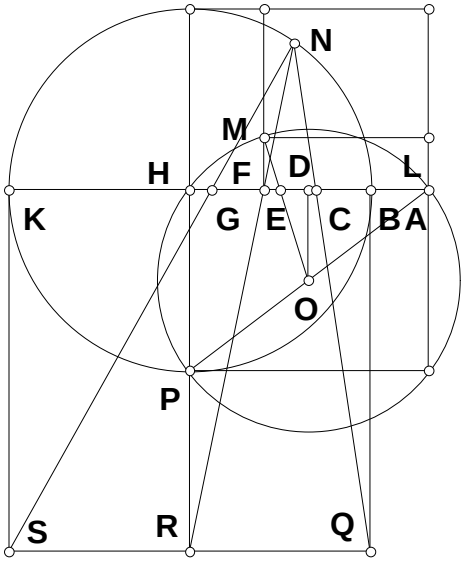
$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$ $AB = 1.523$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

11/18/93 Exploring Cube Roots Plate C

The circle AO passes through point M.



$N := 1$

$BK := 1$ $\quad AB := \frac{BK}{N}$ $\quad AK := AB + BK$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}}$ $\quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$ $\quad \frac{AK}{AB} = 2$

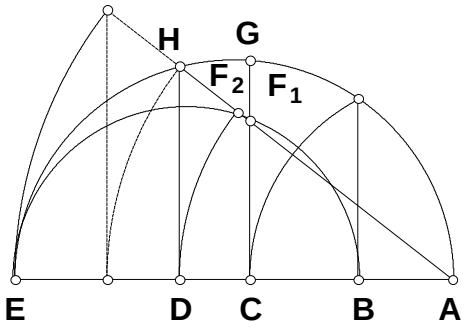
$CG := AG - AC$ $\quad CF := \frac{CG}{2}$ $\quad BH := \frac{BK}{2}$

$AH := AB + BH$ $\quad HP := BH$ $\quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2}$ $\quad DO := \frac{HP}{2}$ $\quad AF := AC + CF$ $\quad AD := \frac{AH}{2}$

$DF := AF - AD$ $\quad FM := CF$ $\quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$



11/22/93 Cube by Iteration

When F_1 and F_2 are the same point on C, then a sixth root series has been constructed. Use iteration to place F_2 on F_1.

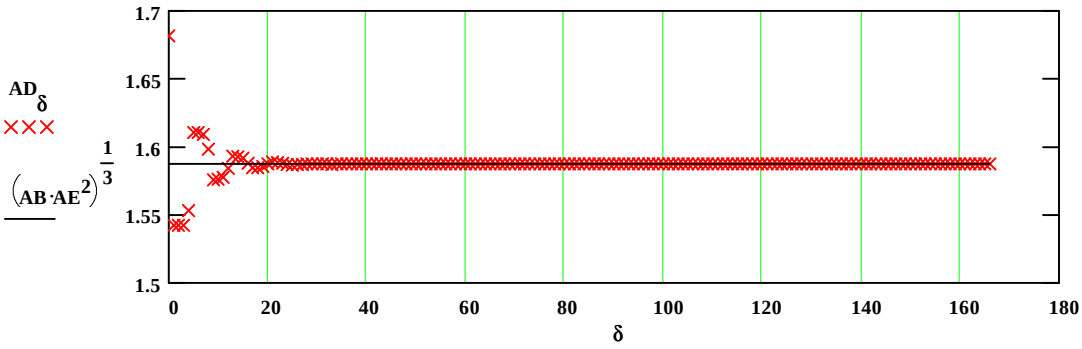
$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[\dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG}\right]^2} \end{bmatrix}$$

$$\begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$

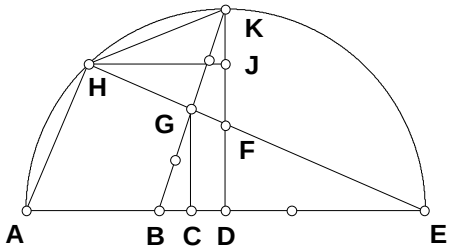
$\Delta \equiv 166$



$$(AB^2 \cdot AE^4)^{\frac{1}{6}} - AD_\Delta = 0 \qquad (AB \cdot AE^2)^{\frac{1}{3}} - AD_\Delta = 0$$

11/24/93 POR Series IV

Generalize the work of 07/25/93 for dividing the base AE with K constant.



$N_1 := 3 \qquad N_2 := 5 \qquad AE := 1$

$\alpha := 1 .. N_1 - 1 \qquad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\frac{AH}{HK} = 0.265 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

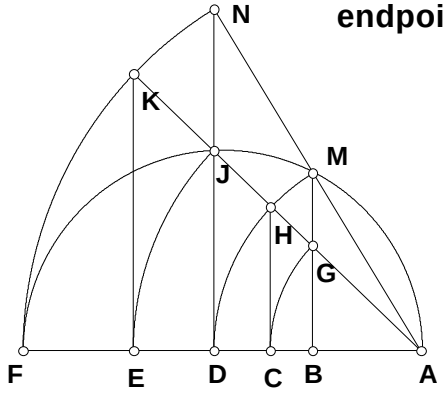
$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$

12/04/93 Exponential Series

Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.



$N_1 := 8$

$AF := N_1 \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

$AN := AF \quad AD := \frac{AB \cdot AN}{AM} \qquad DF := AF - AD$

$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

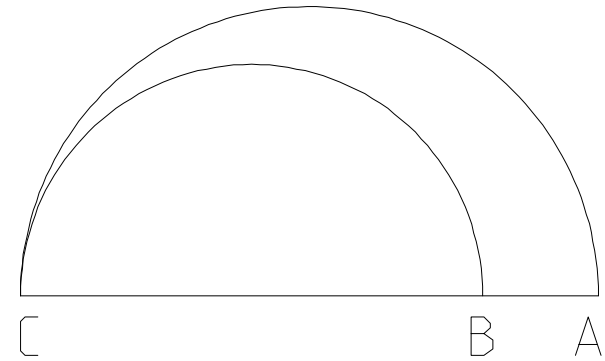
$AK := AF \quad AE := \frac{AD \cdot AK}{AJ} \qquad AH := AD \quad AC := \frac{AD \cdot AH}{AJ}$

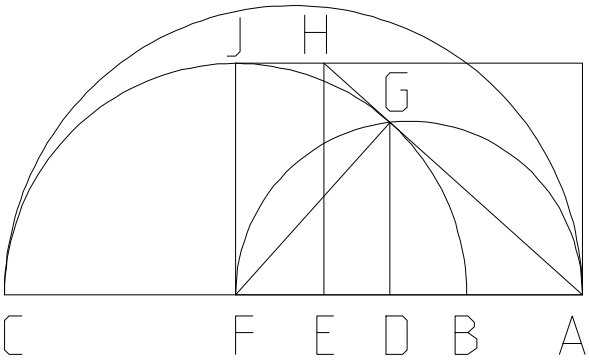
$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$N_1^{\frac{1}{4}} - AC = 0 \qquad N_1^{\frac{2}{4}} - AD = 0 \qquad N_1^{\frac{3}{4}} - AE = 0$

12_06_93

Alternate methods; Square Root .





Demonstrate that the square root of two segments can never be more than one half of the difference between them

Let AB and AC be the two segments and BC the difference between them.

$BC := 10 \quad \delta := 1..100 \quad BF := \frac{BC}{2}$

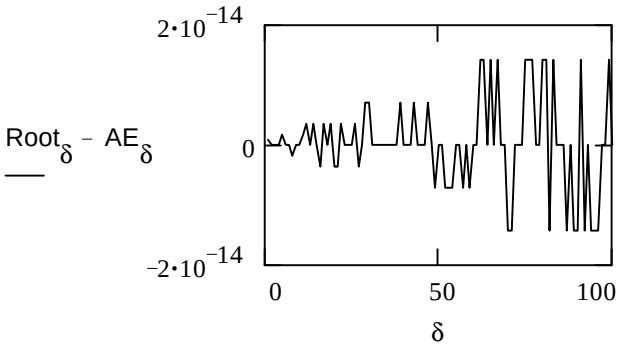
$AB_\delta := \delta \quad AF_\delta := AB_\delta + BF \quad FG := BF$

$DF_\delta := \frac{FG^2}{AF_\delta} \quad FJ := BF \quad AD_\delta := AF_\delta - DF_\delta$

$DG_\delta := \sqrt{AD_\delta \cdot DF_\delta} \quad EH := BF$

$AE_\delta := \frac{AD_\delta \cdot EH}{DG_\delta} \quad AC_\delta := AB_\delta + BC$

$Root_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

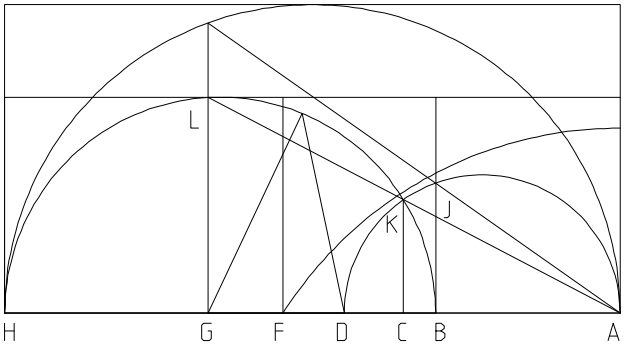


$$\sum_\delta \left(Root_\delta - AE_\delta\right) = 0$$

Since AG can never exceed AF, the result is obvious.

Loging the constant relationship K and J on the cubic.

Gruntwork 4 on the Delian Solution



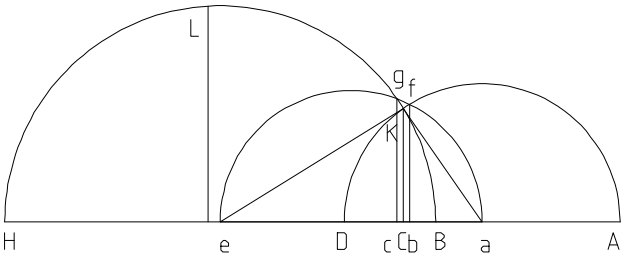
$BH := 10 \quad \delta := 1 .. 1000 \quad AB_\delta := \delta$

$AH_\delta := BH + AB_\delta \quad AF_\delta := \sqrt{AB_\delta \cdot AH_\delta}$

$AG_\delta := \left[\left(AH_\delta\right)^2 \cdot AB_\delta \right]^{\frac{1}{3}}$

$AD_\delta := \left[AH_\delta \cdot \left(AB_\delta\right)^2 \right]^{\frac{1}{3}}$

$BG_\delta := AG_\delta - AB_\delta \quad GH_\delta := BH - BG_\delta \quad GL_\delta := \sqrt{GH_\delta \cdot BG_\delta}$



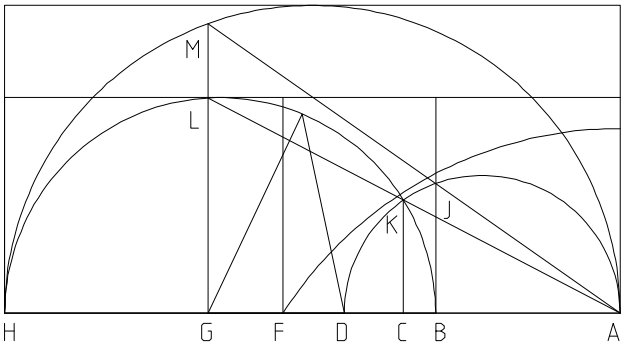
$Aa_\delta := \frac{AD_\delta}{2} \quad Be := \frac{BH}{2} \quad Ka_\delta := Aa_\delta$

$Ke := Be \quad eg := Ke \quad af_\delta := Ka_\delta$

$Ae_\delta := AB_\delta + Be \quad ae_\delta := Ae_\delta - Aa_\delta$

$ce_\delta := \frac{eg^2}{ae_\delta} \quad ab_\delta := \frac{\left(af_\delta\right)^2}{ae_\delta} \quad ac_\delta := ae_\delta - ce_\delta \quad bc_\delta := ac_\delta - ab_\delta \quad Cb_\delta := \frac{bc_\delta}{2}$

$Ca_\delta := ab_\delta + Cb_\delta \quad AC_\delta := Aa_\delta + Ca_\delta \quad Ce_\delta := ae_\delta - Ca_\delta \quad CK_\delta := \sqrt{Ke^2 - \left(Ce_\delta\right)^2}$



$BD_\delta := AD_\delta - AB_\delta$

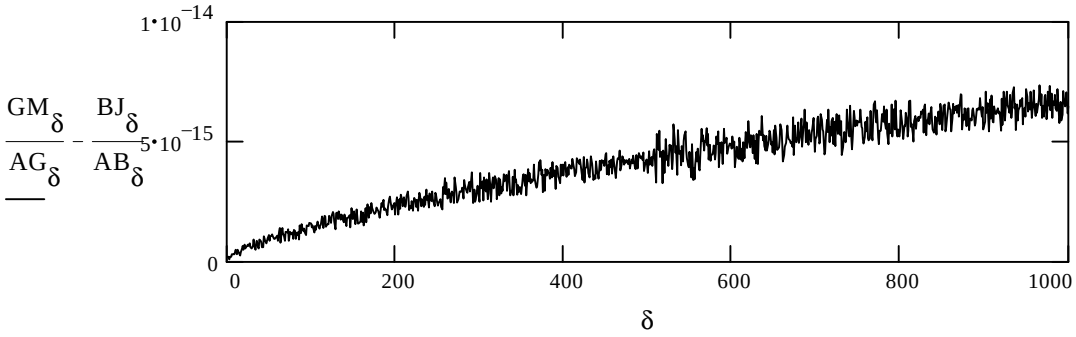
$BJ_\delta := \sqrt{BD_\delta \cdot AB_\delta}$ $GM_\delta := \sqrt{AG_\delta \cdot GH_\delta}$

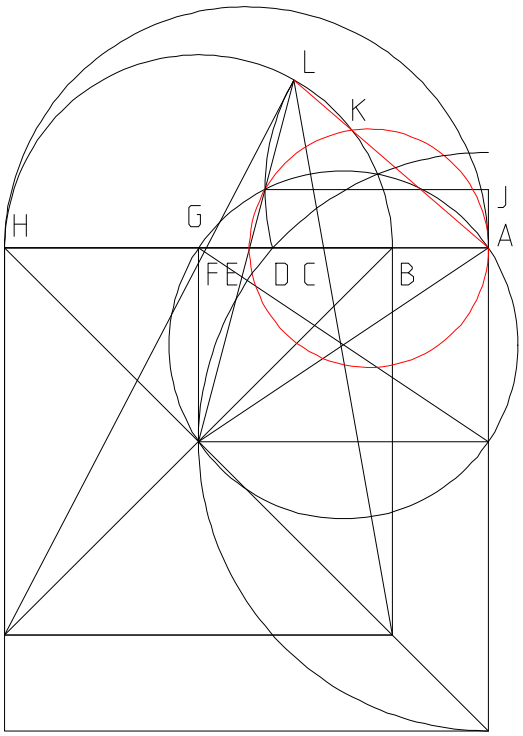
Is K on CK colinear with AL?

$$\sum_\delta \left(\frac{GL_\delta}{AG_\delta} - \frac{CK_\delta}{AC_\delta} \right) = 2.35 \cdot 10^{-13}$$

Is J on BJ colinear with AM?

$$\sum_\delta \left(\frac{GM_\delta}{AG_\delta} - \frac{BJ_\delta}{AB_\delta} \right) = 4.063 \cdot 10^{-12}$$





The structure in red appears to be a constant.

$BH := 5000 \quad \delta := S \,..\, F \quad AB_\delta := \delta^5 \cdot 10^{-8}$

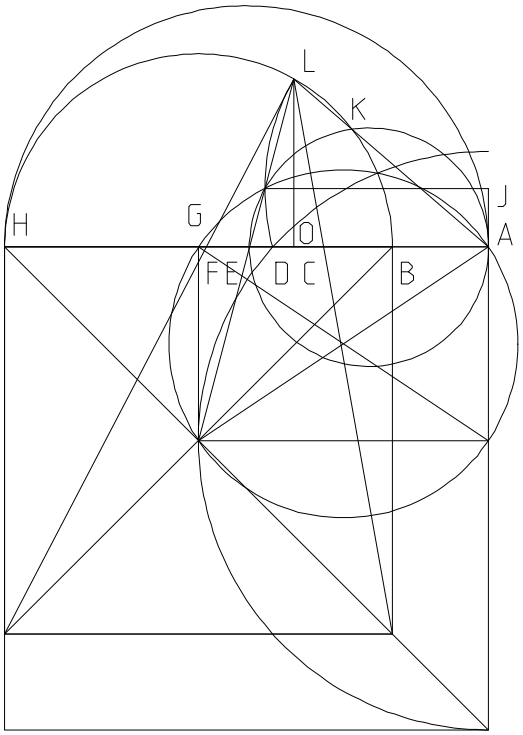
$BG := \frac{BH}{2} \quad AH_\delta := BH + AB_\delta$

$AC_\delta := \left[\left(AB_\delta\right)^2 \cdot AH_\delta \right]^{\frac{1}{3}}$

$AF_\delta := \left[AB_\delta \cdot \left(AH_\delta\right)^2 \right]^{\frac{1}{3}}$

$AJ_\delta := \frac{AF_\delta - AC_\delta}{2} \quad BC_\delta := AC_\delta - AB_\delta$

$AD_\delta := \sqrt{AB_\delta \cdot AH_\delta} \quad BF_\delta := AF_\delta - AB_\delta$



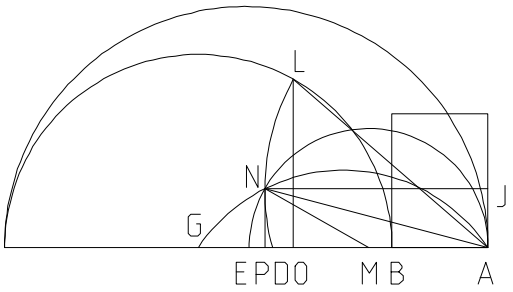
$CF_\delta := AF_\delta - AC_\delta \quad FH_\delta := AH_\delta - AF_\delta$

$CO_\delta := \frac{BC_\delta \cdot CF_\delta}{FH_\delta + BC_\delta} \quad BO_\delta := BC_\delta + CO_\delta$

$GO_\delta := BG - BO_\delta \quad GL := BG$

$LO_\delta := \sqrt{GL^2 - \left(GO_\delta\right)^2}$

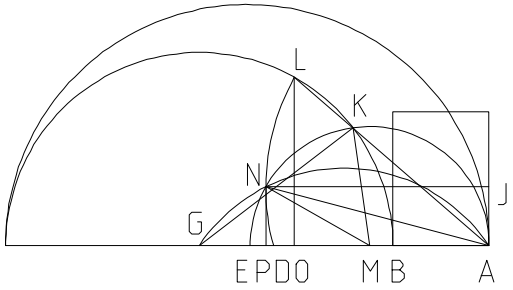
$AO_\delta := AB_\delta + BO_\delta$



$$JD_\delta := \sqrt{(AD_\delta)^2 + (AJ_\delta)^2} \quad JN_\delta := JD_\delta$$

$$NP_\delta := AJ_\delta \quad AN_\delta := \sqrt{(AJ_\delta)^2 + (JN_\delta)^2}$$

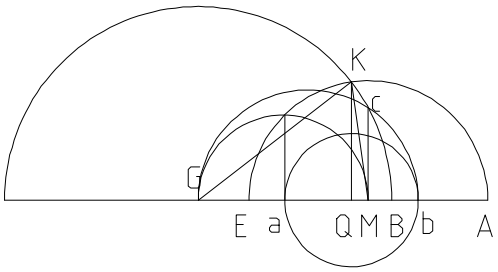
$$AP_\delta := JN_\delta \quad AE_\delta := \frac{(AN_\delta)^2}{AP_\delta}$$



$$AM_\delta := \frac{AE_\delta}{2} \quad MK_\delta := AM_\delta \quad GK := BG$$

$$AG_\delta := AB_\delta + BG$$

$$GM_\delta := AG_\delta - AM_\delta \quad Ma_\delta := \frac{(MK_\delta)^2}{GM_\delta}$$

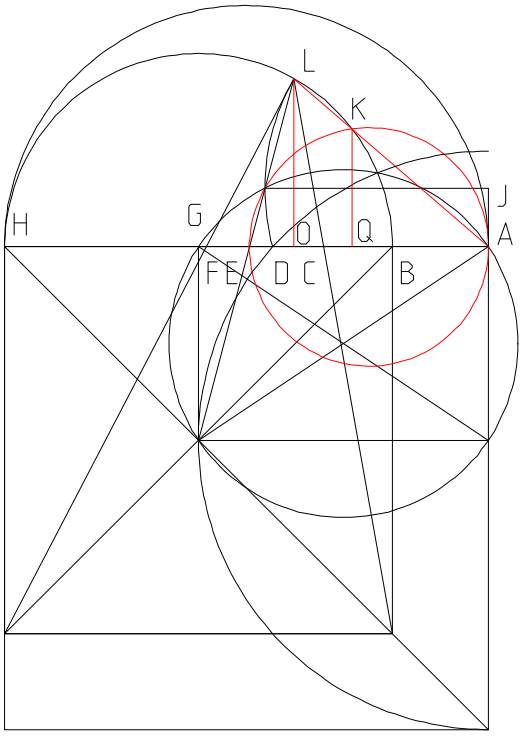


$$Gb_\delta := \frac{(GK)^2}{GM_\delta} \quad Ga_\delta := GM_\delta - Ma_\delta$$

$$ab_\delta := Gb_\delta - Ga_\delta \quad Qa_\delta := \frac{ab_\delta}{2}$$

$$GQ_\delta := Ga_\delta + Qa_\delta \quad AQ_\delta := AG_\delta - GQ_\delta$$

$$KQ_\delta := \sqrt{GK^2 - (GQ_\delta)^2}$$



$\left(AB_S\right)^2 \cdot AH_S = 4.886 \cdot 10^4$ $BO_S = 36.27$

$AB_S \cdot \left(AH_S\right)^2 = 7.822 \cdot 10^7$ $BO_F = 109.49532$

$\left(AB_F\right)^2 \cdot AH_F = 1.417 \cdot 10^6$ $AB_S = 3.125$

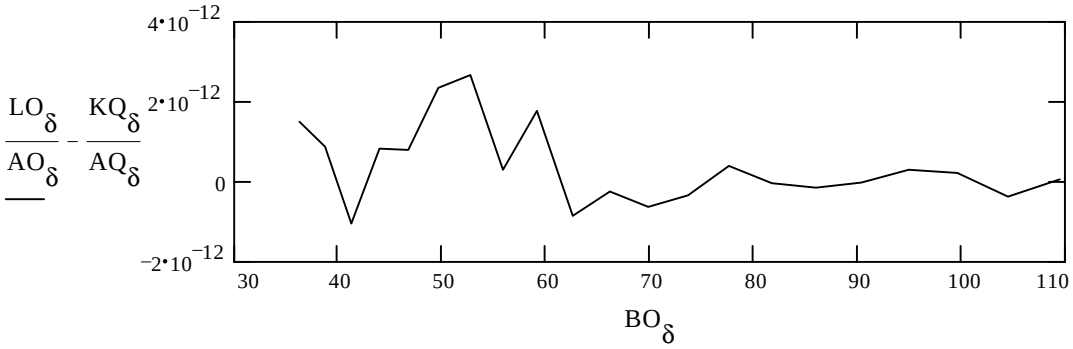
$AB_F \cdot \left(AH_F\right)^2 = 4.23 \cdot 10^8$ $AB_F = 16.807$

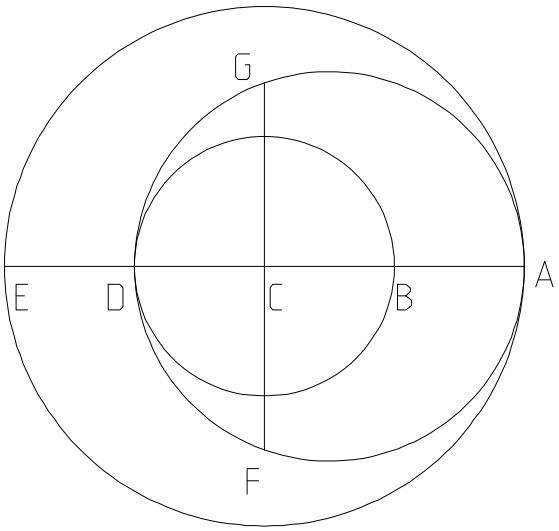
$LO_S = 424.30457$ $KQ_S = 36.53305$

$LO_F = 731.77$ $KQ_F = 111.891$

$$\sum_{\delta}\left(\frac{LO_\delta}{AO_\delta} - \frac{KQ_\delta}{AQ_\delta}\right) = 8.477 \cdot 10^{-12}$$

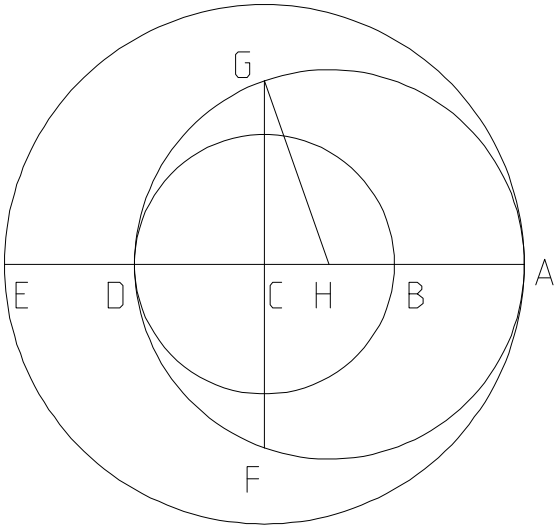
$S \equiv 50$ $F \equiv 70$





The Square Root

It may be noticed that I use the adjacent figure in my work for doing square roots. I believe that it is the primary figure for doing square roots. Given segment AE and segment BD, segment FG is their square root. It has a major advantage of being able to square larger figures on paper, not to mention makes something of the development of exponential series.



$AE := 100 \quad \delta := 1 .. AE \quad BD_\delta := \delta$

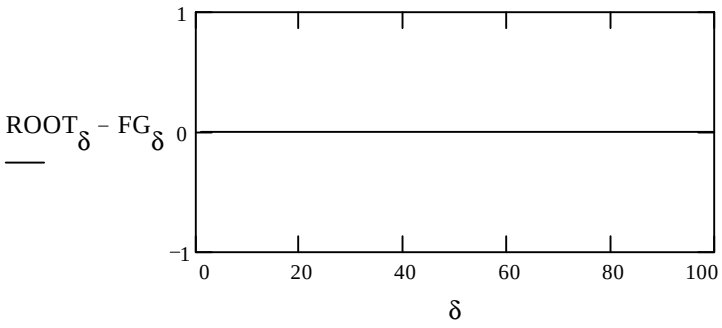
$AC := \frac{AE}{2} \quad BC_\delta := \frac{BD_\delta}{2} \quad AB_\delta := AC - BC_\delta$

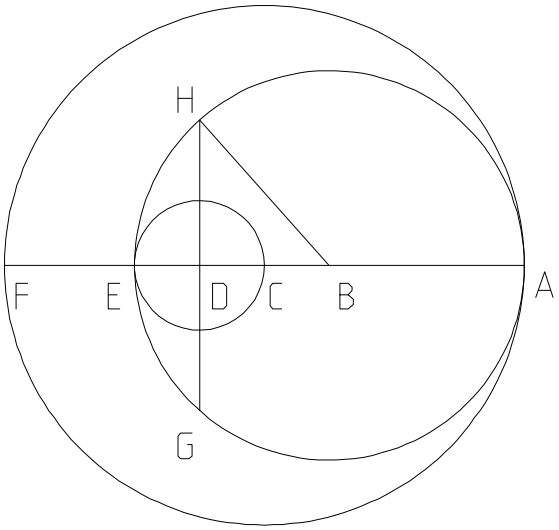
$AD_\delta := AC + BC_\delta \quad AH_\delta := \frac{AD_\delta}{2}$

$GH_\delta := AH_\delta \quad CH_\delta := AC - AH_\delta$

$FG_\delta := 2 \cdot \sqrt{(GH_\delta)^2 - (CH_\delta)^2}$

$ROOT_\delta := \sqrt{(BD_\delta) \cdot AE}$





12_12_B3.MCD

Completely generalize the square root figure.

$AF := 7 \qquad DF := \frac{AF}{BR} \qquad AD := AF - DF$

$\delta := 1 .. LBR \qquad DE_\delta := \frac{DF}{LBR} \cdot \delta$

$CE_\delta := 2 \cdot DE_\delta \qquad AE_\delta := AD + DE_\delta$

$AB_\delta := \frac{AE_\delta}{2} \qquad BD_\delta := AD - AB_\delta$

$BH_\delta := AB_\delta \qquad GH_\delta := 2 \cdot \sqrt{\left| (BH_\delta)^2 - (BD_\delta)^2 \right|}$

Set AF to unity so that it may be eliminated. Setting BR to 2 will yeild the familiar square root. BR may even take fractional values.

Plug in values here.

BR=BASE RATIO,
LBR=LITTLE BASE RATIO

$BR \equiv 3 \qquad LBR \equiv 5$

$\left| \frac{(2 \cdot BR) - 2}{BR} \right| = 1.333$

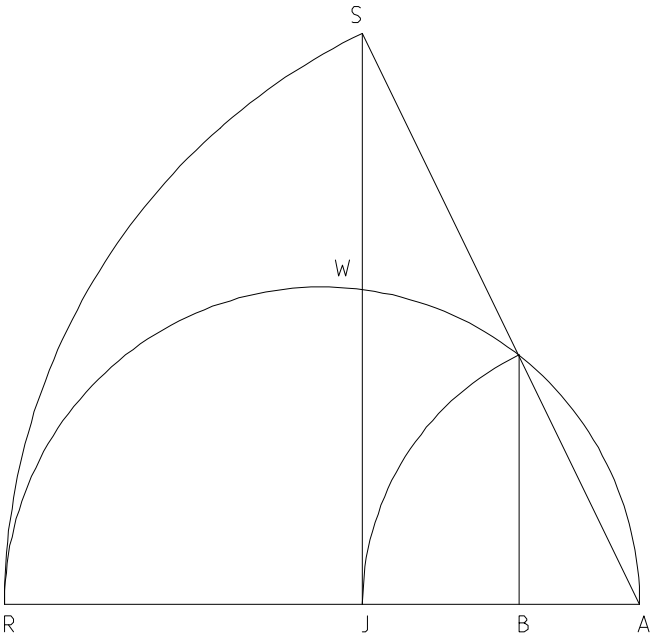
The equation below simplifies to the next.

$GH_\delta \qquad \left[\frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot \left| \frac{(2 \cdot BR) - 2}{BR} \right| \right]^{\frac{1}{2}} \cdot AF \qquad \frac{\sqrt{2} \cdot \sqrt{\left| (2 \cdot BR) - 2 \right|}}{BR \cdot \sqrt{LBR}} \cdot \sqrt{\delta} \cdot AF$

2.951	2.951	2.951
4.174	4.174	4.174
5.112	5.112	5.112
5.903	5.903	5.903
6.6	6.6	6.6

$CE_\delta \qquad \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot AF$

0.933	0.933
1.867	1.867
2.8	2.8
3.733	3.733
4.667	4.667



12_16_93, using 12_04_93.MCD

$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

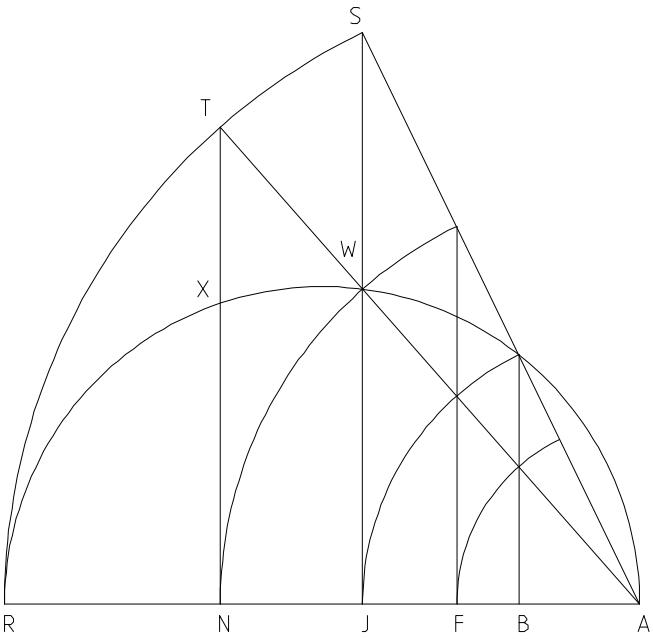
$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

AB_δ	AJ_δ
5	7.071
3.333	5.774
2.5	5
2	4.472
1.667	4.082

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.

Euclidean Exponential Series



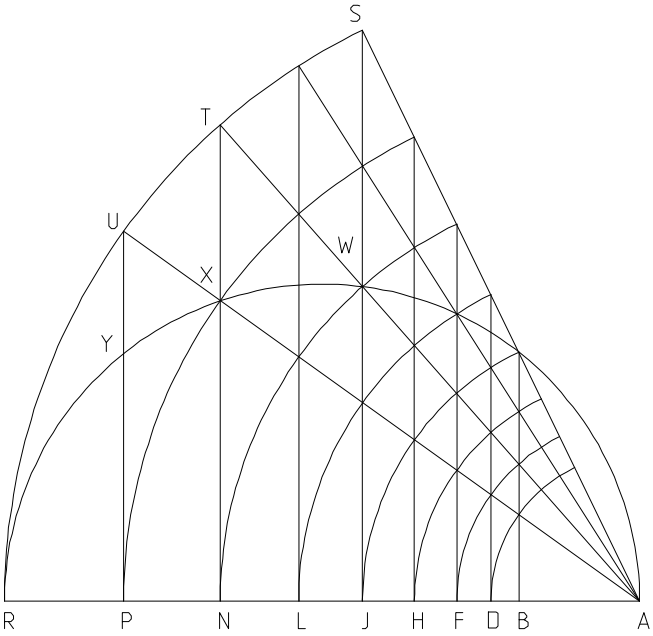
$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

AB_δ	AF_δ	AJ_δ	AN_δ
5	5.946	7.071	8.409
3.333	4.387	5.774	7.598
2.5	3.536	5	7.071
2	2.991	4.472	6.687
1.667	2.608	4.082	6.389

What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.



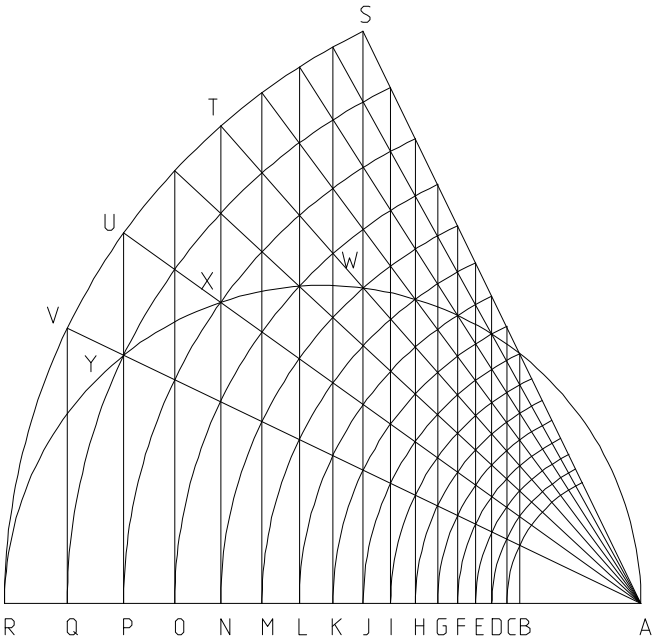
$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$

$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$

$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$

$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$

AB_δ	AD_δ	AF_δ	AH_δ	AJ_δ	AL_δ	AN_δ	AP_δ
5	5.453	5.946	6.484	7.071	7.711	8.409	9.17
3.333	3.824	4.387	5.033	5.774	6.623	7.598	8.717
2.5	2.973	3.536	4.204	5	5.946	7.071	8.409
2	2.446	2.991	3.657	4.472	5.469	6.687	8.178
1.667	2.085	2.608	3.263	4.082	5.107	6.389	7.993



$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$

$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$

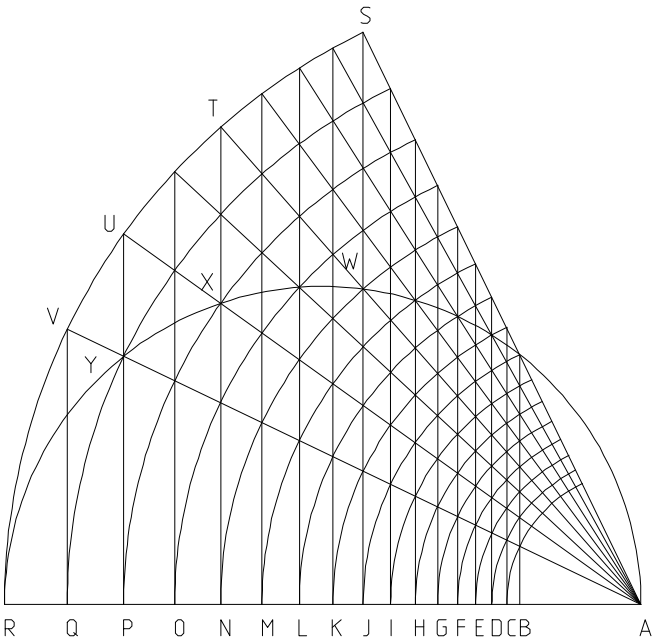
$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$

$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$

AB_δ	AC_δ	AD_δ	AE_δ	AF_δ
5	5.221	5.453	5.694	5.946
3.333	3.57	3.824	4.096	4.387
2.5	2.726	2.973	3.242	3.536
2	2.212	2.446	2.704	2.991
1.667	1.864	2.085	2.332	2.608

AG_δ	AH_δ	AI_δ	AJ_δ	AK_δ	AL_δ	AM_δ	AN_δ	AO_δ	AP_δ	AQ_δ
6.209	6.484	6.771	7.071	7.384	7.711	8.052	8.409	8.781	9.17	9.576
4.699	5.033	5.39	5.774	6.184	6.623	7.094	7.598	8.138	8.717	9.336
3.856	4.204	4.585	5	5.453	5.946	6.484	7.071	7.711	8.409	9.17
3.307	3.657	4.044	4.472	4.945	5.469	6.047	6.687	7.395	8.178	9.043
2.918	3.263	3.65	4.082	4.566	5.107	5.713	6.389	7.147	7.993	8.941

Values found by the investigator of 12_14_93



AQ_δ	$\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$
9.576	9.576
9.336	9.336
9.17	9.17
9.043	9.043
8.941	8.941

AP_δ	$\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$
9.17	9.17
8.717	8.717
8.409	8.409
8.178	8.178
7.993	7.993

AN_δ	$\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$
8.409	8.409
7.598	7.598
7.071	7.071
6.687	6.687
6.389	6.389

AL_δ	$\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$
7.711	7.711
6.623	6.623
5.946	5.946
5.469	5.469
5.107	5.107

AI_δ	$\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$
6.771	6.771
5.39	5.39
4.585	4.585
4.044	4.044
3.65	3.65

AH_δ	$\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$
6.484	6.484
5.033	5.033
4.204	4.204
3.657	3.657
3.263	3.263

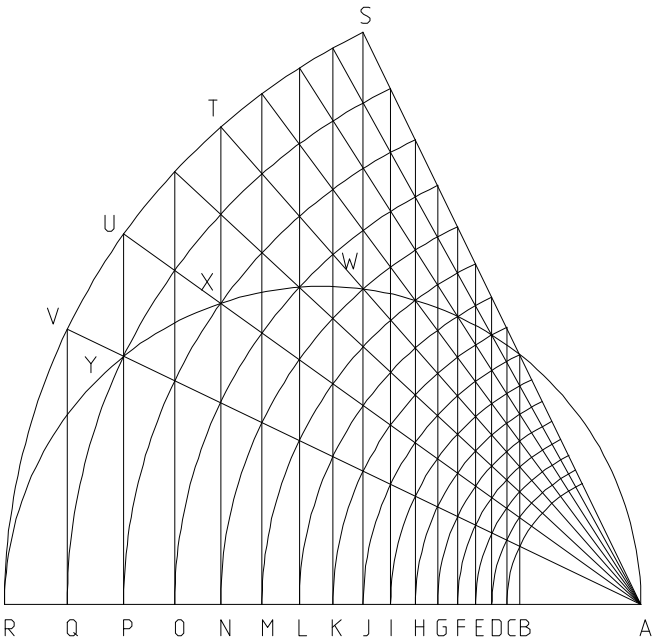
AG_δ	$\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$
6.209	6.209
4.699	4.699
3.856	3.856
3.307	3.307
2.918	2.918

AF_δ	$\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$
5.946	5.946
4.387	4.387
3.536	3.536
2.991	2.991
2.608	2.608

AE_δ	$\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$
5.694	5.694
4.096	4.096
3.242	3.242
2.704	2.704
2.332	2.332

AD_δ	$\left[\left(AB_\delta\right)^{7} \cdot AR\right]^{\frac{1}{8}}$
5.453	5.453
3.824	3.824
2.973	2.973
2.446	2.446
2.085	2.085

AC_δ	$\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$
5.221	5.221
3.57	3.57
2.726	2.726
2.212	2.212
1.864	1.864



AQ_δ	$\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$
9.576	9.576
9.336	9.336
9.17	9.17
9.043	9.043
8.941	8.941

AP_δ	$\left[\left(AB_\delta\right)^2 \cdot AR^{14}\right]^{\frac{1}{16}}$
9.17	9.17
8.717	8.717
8.409	8.409
8.178	8.178
7.993	7.993

AO_δ	$\left[\left(AB_\delta\right)^3 \cdot AR^{13}\right]^{\frac{1}{16}}$
8.781	8.781
8.138	8.138
7.711	7.711
7.395	7.395
7.147	7.147

AN_δ	$\left[\left(AB_\delta\right)^4 \cdot AR^{12}\right]^{\frac{1}{16}}$
8.409	8.409
7.598	7.598
7.071	7.071
6.687	6.687
6.389	6.389

AM_δ	$\left[\left(AB_\delta\right)^5 \cdot AR^{11}\right]^{\frac{1}{16}}$
8.052	8.052
7.094	7.094
6.484	6.484
6.047	6.047
5.713	5.713

AL_δ	$\left[\left(AB_\delta\right)^6 \cdot AR^{10}\right]^{\frac{1}{16}}$
7.711	7.711
6.623	6.623
5.946	5.946
5.469	5.469
5.107	5.107

AK_δ	$\left[\left(AB_\delta\right)^7 \cdot AR^{9}\right]^{\frac{1}{16}}$
7.384	7.384
6.184	6.184
5.453	5.453
4.945	4.945
4.566	4.566

AJ_δ	$\left[\left(AB_\delta\right)^8 \cdot AR^{8}\right]^{\frac{1}{16}}$
7.071	7.071
5.774	5.774
5	5
4.472	4.472
4.082	4.082

AI_δ	$\left[\left(AB_\delta\right)^9 \cdot AR^{7}\right]^{\frac{1}{16}}$
6.771	6.771
5.39	5.39
4.585	4.585
4.044	4.044
3.65	3.65

AH_δ	$\left[\left(AB_\delta\right)^{10} \cdot AR^{6}\right]^{\frac{1}{16}}$
6.484	6.484
5.033	5.033
4.204	4.204
3.657	3.657
3.263	3.263

AG_δ	$\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$
6.209	6.209
4.699	4.699
3.856	3.856
3.307	3.307
2.918	2.918

AF_δ	$\left[\left(AB_\delta\right)^{12} \cdot AR^{4}\right]^{\frac{1}{16}}$
5.946	5.946
4.387	4.387
3.536	3.536
2.991	2.991
2.608	2.608

AE_δ	$\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$
5.694	5.694
4.096	4.096
3.242	3.242
2.704	2.704
2.332	2.332

AD_δ	$\left[\left(AB_\delta\right)^{14} \cdot AR^{2}\right]^{\frac{1}{16}}$
5.453	5.453
3.824	3.824
2.973	2.973
2.446	2.446
2.085	2.085

AC_δ	$\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$
5.221	5.221
3.57	3.57
2.726	2.726
2.212	2.212
1.864	1.864

AB_δ	$\left[\left(AB_\delta\right)^{16} \cdot AR^{0}\right]^{\frac{1}{16}}$
5	5
3.333	3.333
2.5	2.5
2	2
1.667	1.667

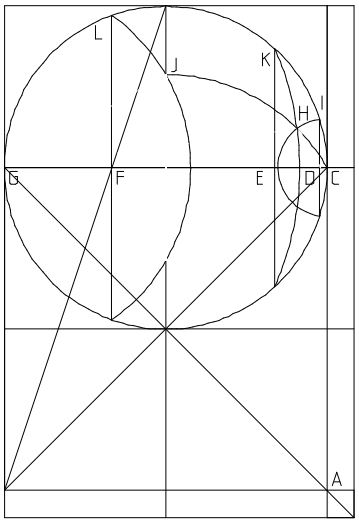
$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

Resultant Equation

depending on direction of transcription.



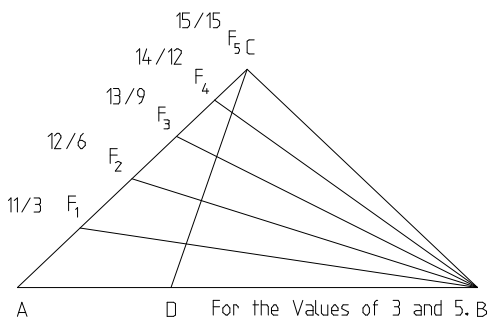
One Circle

By John Clark

One Square

One Line

And the Delian Quest



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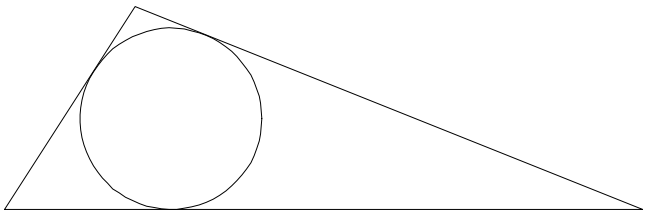
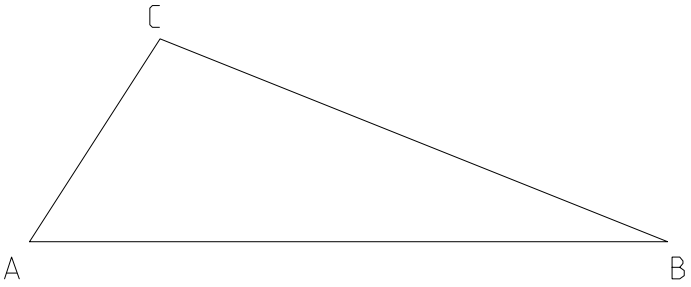


Inscribing a Circle in a given Triangle.

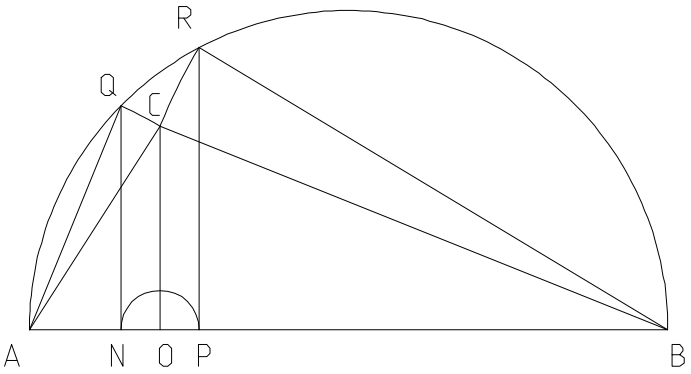
Place the length for the sides of the triangle at the end of the document.



$$AB := \begin{pmatrix} Side_1 \\ Side_2 \\ Side_3 \end{pmatrix} \quad AC := \begin{pmatrix} Side_2 \\ Side_3 \\ Side_1 \end{pmatrix}$$

$$BC := \begin{pmatrix} Side_3 \\ Side_1 \\ Side_2 \end{pmatrix} \quad \delta := 0..2$$

$$Is_This_A_Triangle := \left(AB_0 + AC_0 \geq BC_0\right) \cdot \left(AB_0 + BC_0 \geq AC_0\right) \cdot \left(AC_0 + BC_0 \geq AB_0\right) \quad AQ_\delta := AC_\delta$$



$$BR_\delta := BC_\delta \quad AN_\delta := \frac{\left(AQ_\delta\right)^2}{AB_\delta}$$

$$BP_\delta := \frac{\left(BR_\delta\right)^2}{AB_\delta} \quad AP_\delta := AB_\delta - BP_\delta$$

$$NP_\delta := AP_\delta - AN_\delta \quad NO_\delta := \frac{NP_\delta}{2}$$

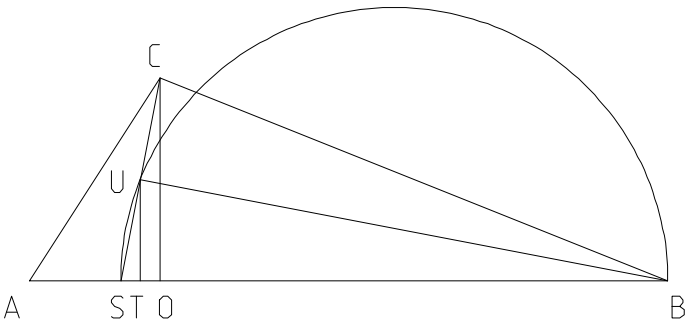
$$AO := AN + NO \quad BO_\delta := AB_\delta - AO_\delta$$

$$CO_\delta := \sqrt{\left(BC_\delta\right)^2 - \left(BO_\delta\right)^2}$$

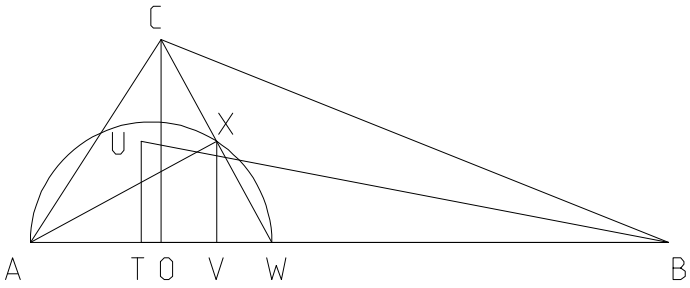
$$BS_\delta := BC_\delta \quad SO_\delta := BS_\delta - BO_\delta$$

$$CS_\delta := \sqrt{\left(SO_\delta\right)^2 + \left(CO_\delta\right)^2} \quad SU_\delta := \frac{CS_\delta}{2}$$

$$BU_\delta := \sqrt{\left(BS_\delta\right)^2 - \left(SU_\delta\right)^2}$$



$$ST_\delta := \frac{\left(SU_\delta\right)^2}{BS_\delta} \quad TU_\delta := \sqrt{\left(SU_\delta\right)^2 - \left(ST_\delta\right)^2}$$

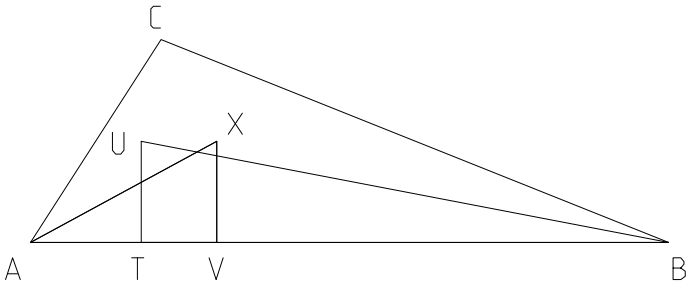


$$AW_\delta := AC_\delta \quad WO_\delta := AW_\delta - AO_\delta$$

$$CW_\delta := \sqrt{(WO_\delta)^2 + (CO_\delta)^2}$$

$$WX_\delta := \frac{CW_\delta}{2}$$

$$AX_\delta := \sqrt{(AW_\delta)^2 - (WX_\delta)^2}$$

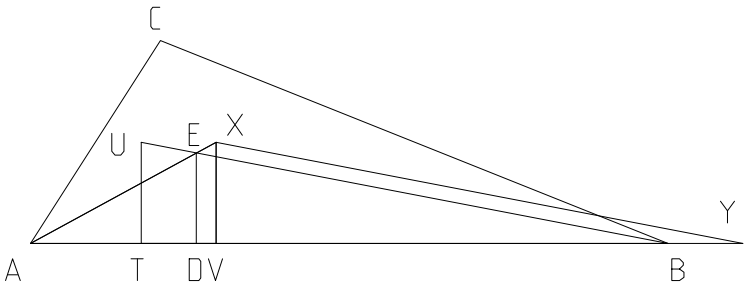


$$WV_\delta := \frac{(WX_\delta)^2}{AW_\delta}$$

$$VX_\delta := \sqrt{(WX_\delta)^2 - (WV_\delta)^2}$$

$$WV_\delta := \sqrt{(WX_\delta)^2 - (VX_\delta)^2}$$

$$AV_\delta := AW_\delta - WV_\delta \quad BV_\delta := AB_\delta - AV_\delta \quad XY_\delta := \frac{BU_\delta \cdot VX_\delta}{TU_\delta} \quad VY_\delta := \sqrt{(XY_\delta)^2 - (VX_\delta)^2}$$



$$AY_\delta := AV_\delta + VY_\delta$$

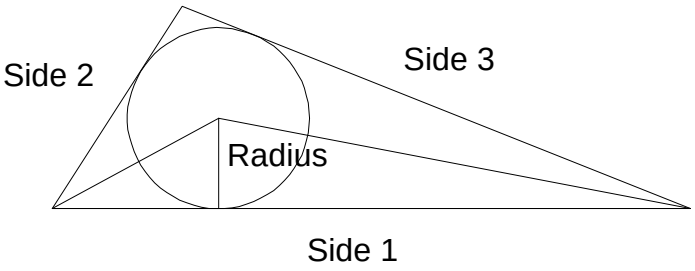
$$AD_\delta := \frac{AV_\delta \cdot AB_\delta}{AY_\delta} \quad AE_\delta := \frac{AX_\delta \cdot AD_\delta}{AV_\delta}$$

$$DE_\delta := \sqrt{(AE_\delta)^2 - (AD_\delta)^2}$$

Plug Side Values In Here $Side_1 \equiv 20$ $Side_2 \equiv 26$ $Side_3 \equiv 21$

$AB^T = (20 \quad 26 \quad 21)$ $Is_This_A_Triangle = 1$

Given 3 lengths, no matter what order they are entered, DE should remain a constant.

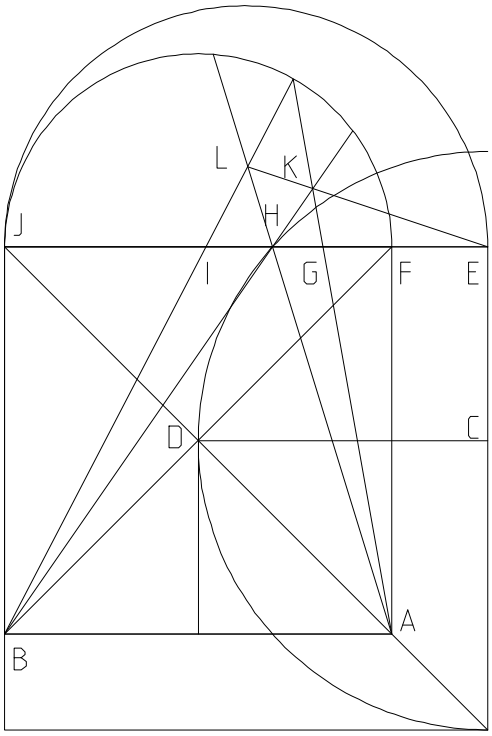


$S_1 := AB \quad S_2 := AC \quad S_3 := BC$

You will note that the formula derived from the process is more consistent with imaginaries.

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}{2\cdot\sqrt{S1_\delta + S2_\delta + S3_\delta}}$$

$$Radius = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix} \qquad DE = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix}$$



The Cradle

Is EL and EK always collinear?

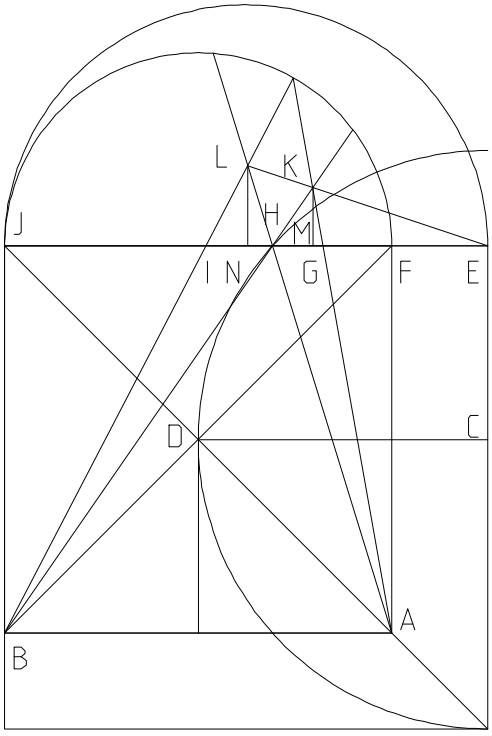
$l := 100 \qquad \delta := 1 .. l \qquad EF_\delta := \delta^5 \cdot 10^{-8}$

$FJ := 10 \qquad EJ_\delta := FJ + EF_\delta$

$EH_\delta := \sqrt{EF_\delta \cdot EJ_\delta} \qquad FH_\delta := EH_\delta - EF_\delta$

$EG_\delta := \left[\left(EF_\delta \right)^2 \cdot EJ_\delta \right]^{\frac{1}{3}} \qquad FG_\delta := EG_\delta - EF_\delta$

$EI_\delta := \left[\left(EJ_\delta \right)^2 \cdot EF_\delta \right]^{\frac{1}{3}} \qquad FI_\delta := EI_\delta - EF_\delta$



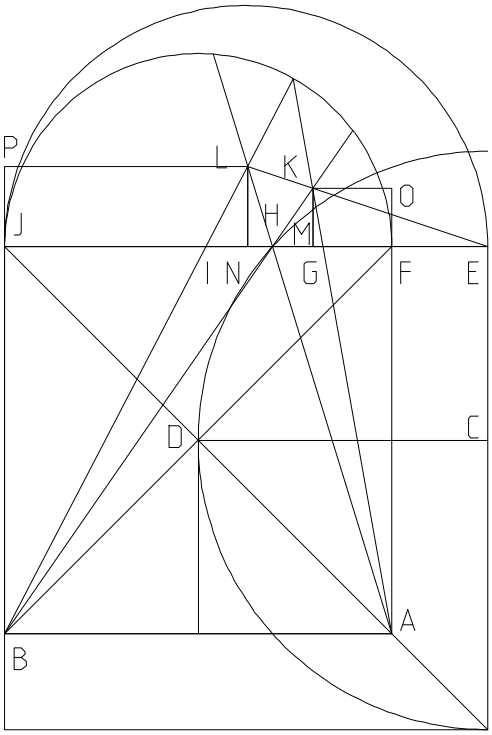
Basically it is demonstrated that the two triangles EKM and ELN are proportional, which is sufficient.

$IJ_\delta := FJ - FI_\delta$

$HJ_\delta := FJ - FH_\delta$

$FN_\delta := \frac{FH_\delta \cdot FJ}{\left(FH_\delta + IJ_\delta \right)} \qquad EN_\delta := FN_\delta + EF_\delta$

$FM_\delta := \frac{FG_\delta \cdot FJ}{\left(FG_\delta + HJ_\delta \right)} \qquad EM_\delta := FM_\delta + EF_\delta$



$$JN_\delta := FJ - FN_\delta \quad LP_\delta := JN_\delta$$

$$BJ := FJ \quad BP_\delta := \frac{BJ \cdot LP_\delta}{IJ_\delta}$$

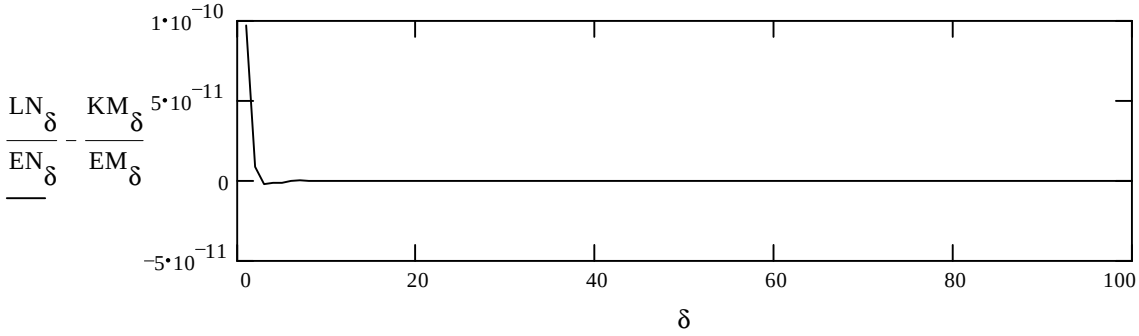
$$JP_\delta := BP_\delta - BJ \quad LN_\delta := JP_\delta$$

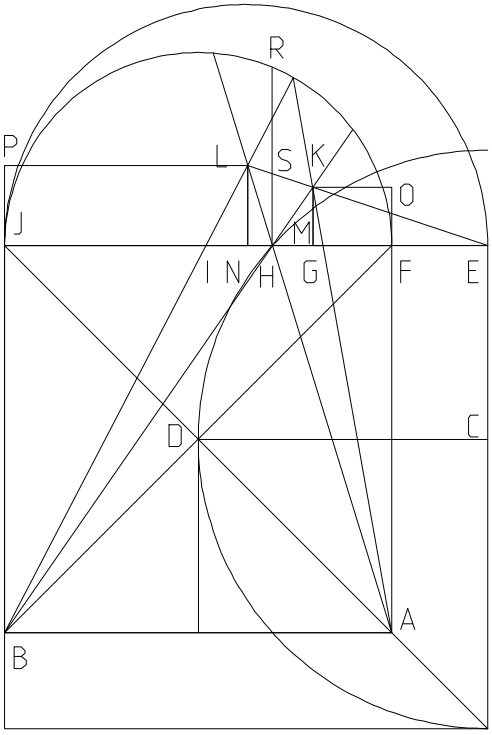
$$KO_\delta := FM_\delta \quad AF := FJ$$

$$AO_\delta := \frac{AF \cdot KO_\delta}{FG_\delta} \quad FO_\delta := AO_\delta - AF$$

$$KM_\delta := FO_\delta \qquad \sum_\delta \left(\frac{LN_\delta}{EN_\delta} - \frac{KM_\delta}{EM_\delta} \right) = 1.015 \cdot 10^{-10}$$

They are two lines with identical slopes, terminating at the same point.

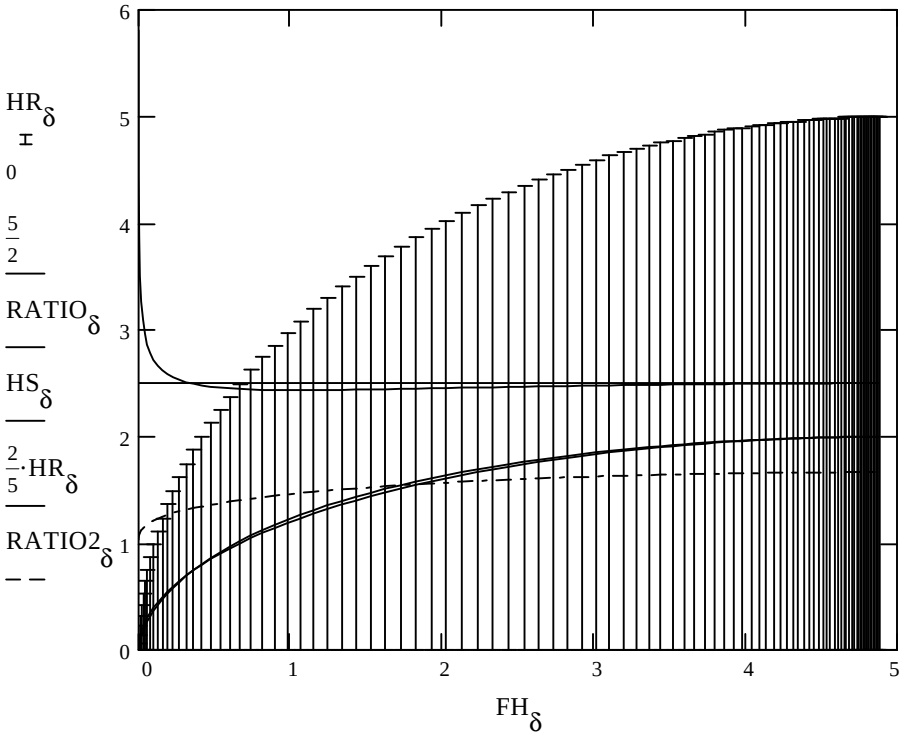




$$HR_\delta := \sqrt{FH_\delta \cdot HJ_\delta} \qquad HS_\delta := \frac{LN_\delta \cdot EH_\delta}{EN_\delta}$$

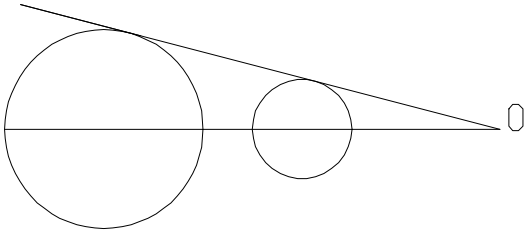
$$RATIO_\delta := \frac{HR_\delta}{HS_\delta} \qquad GI_\delta := EI_\delta - EG_\delta$$

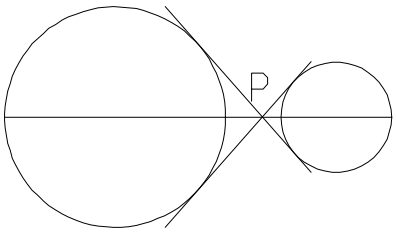
$$RATIO2_\delta := \frac{GI_\delta}{HS_\delta}$$

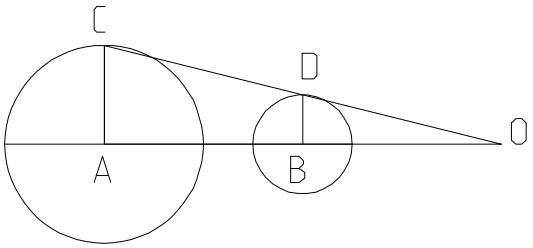


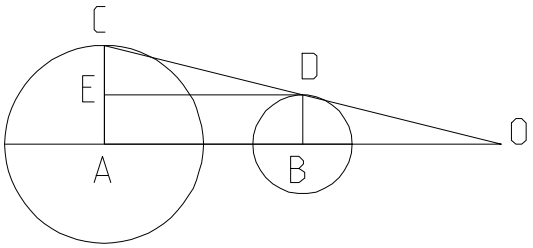
Tangents and Similarity Points

04_26_94.MCD









O and P are points of origin for the ratio of the two circles that can also have a tangent ray to both circles. Develop formulas that would locate the particular points given using just the radius of the two circles and the difference between them.

O and P are called the similarity points (sp) of the two circles. O is the external similarity point and P is the internal similarity point.

I will work with point O first.

Given R_L = large radius

R_S = small radius

D = difference between origins.

$R_L := 4 \quad R_S := 1 \quad D := 8$

$AC := R_L \quad BD := R_S \quad AB := D$

If the difference between the circles is less than R_L-R_S, than one of course has an imaginary situation for the external similarity point, $R_L + R_S$ for the internal. At R_L-R_S the smaller is in the larger and they touch at one point, $R_L + R_S$ they are external to one another and touching.

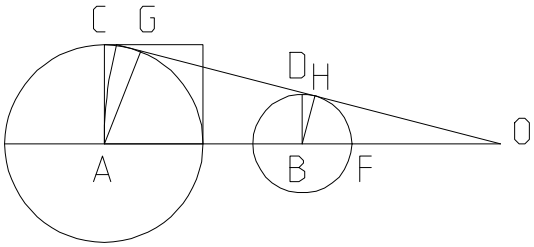
$DE := AB \quad AE := BD \quad CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE} \quad AO = 10.667$

EOR_L "External similarity point Origin to center of Radius Large"

$$EOR_L := if\left(R_L \neq R_S, if\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right)$$

$EOR_L = 10.667$



What is the length of line (OG) tangent to both circles?

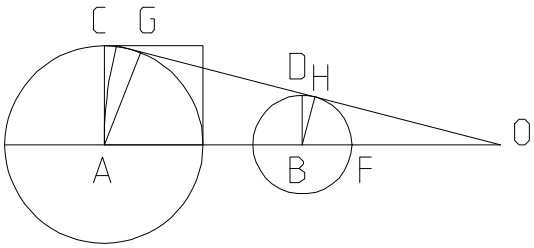
$AG := AC \quad GO := \sqrt{AO^2 - AG^2}$

$GO = 9.888$

And what is the formula?
EOT_{LR} " External similarity point Origin to Tangent (Large Radius)"

$$EOT_{LR} := R_L \cdot \frac{\sqrt{\left(R_L - R_S + D\right)\cdot\left(-R_L + R_S + D\right)}}{R_L - R_S}$$

$EOT_{LR} = 9.888$



What is the length of the line tangent to the least circle (HO)?

$BH := BD \quad BO := AO - AB \qquad BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

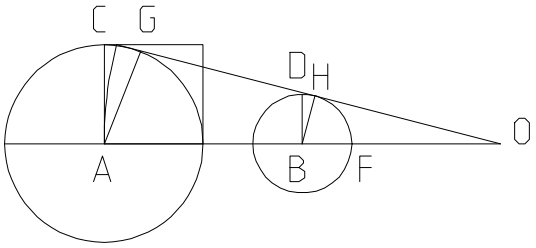
$HO = 2.472$

And what is the formula?

EOT_{SR} " External similarity point Origin to Tangent (Small Radius)"

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}}{R_L - R_S}$$

$EOT_{SR} = 2.472$



Lastly what is the length of line from tangent to tangent of these circles?

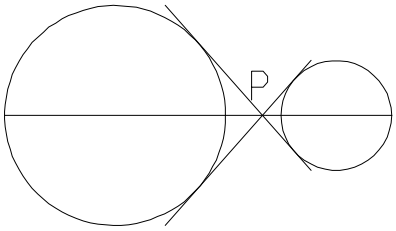
$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

And what is the formula?
ETT "Tangent to Tangent"

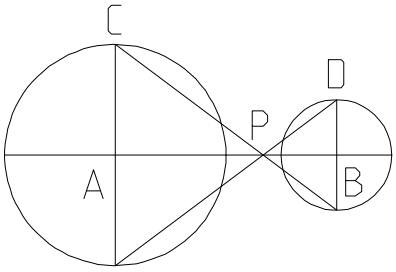
$$ETT := \sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}$$

$ETT = 7.416$



I will now turn my attention to the point P, the internal similarity point.

$$AP := \frac{AB\cdot AC}{AC + BD} \qquad AP = 6.4$$



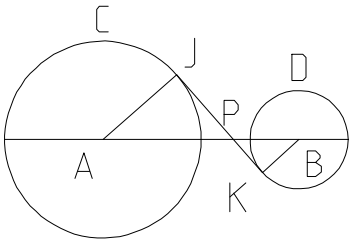
IOR_L "Internal similarity point to center of Radius Large"

$$IOR_L := D\cdot\frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

IOR_S "Internal similarity point to center of Radius Small"

$$IOR_S := D\cdot\frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$



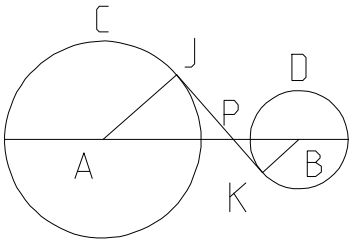
$AJ := AC \quad BK := BD \quad JP := \sqrt{AP^2 - AJ^2}$

$JP = 4.996$

IOT_{LR} "Internal similarity point Origin to Tangent (Large Radius)"

$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996$

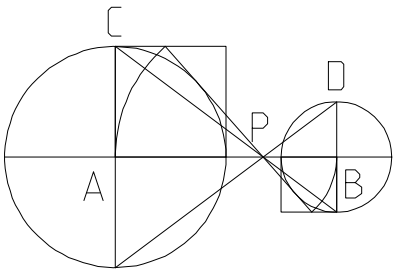


$KP := \sqrt{BP^2 - BK^2} \quad KP = 1.249$

IOT_{SR} "Internal similarity point Origin to Tangent (Small Radius)"

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249$



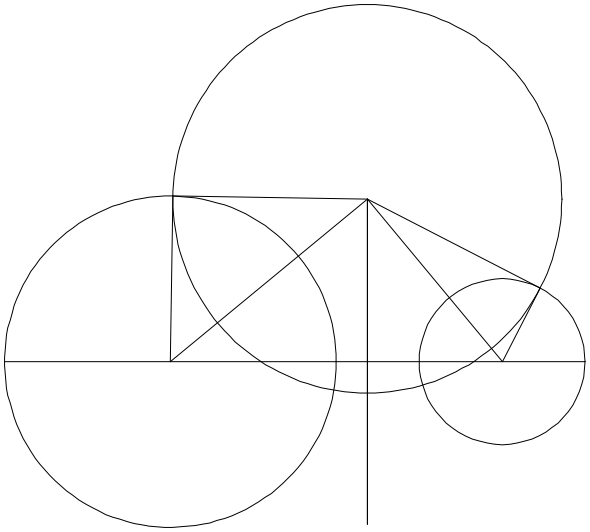
$JK := JP + KP \quad JK = 6.245$

ITT "Internal similarity point Tangent to Tangent"

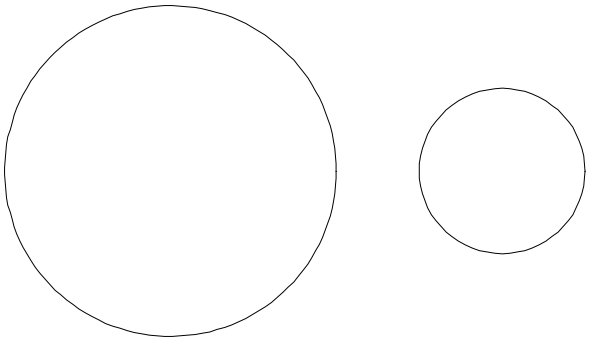
$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}$$

$ITT = 6.245$

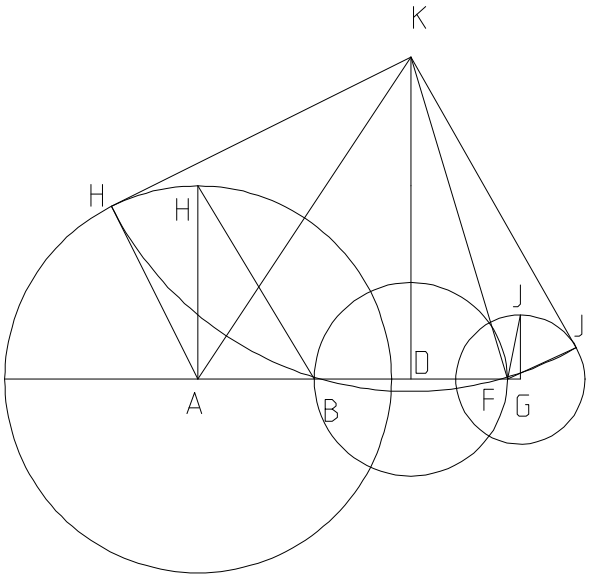
The Chordal or Power Line of two Circles 04_27_94.MCD



The solution for finding the chordal as outlined in ***100 Great Problems of Elementary Mathematics Their History and Solution*** by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.
The figure I work with is a transformation of the one on the left.



Given two circles find their chordal or power line given just their radius and difference between their centers, and reduce the tautological chains to formulas.



$R_1 := 3 \quad R_2 := 1 \quad D := 1$

$AH := R_1 \quad GJ := R_2 \quad AG := D$

$AB := \frac{AH^2}{AG} \quad FG := \frac{GJ^2}{AG}$

$BF := AG - AB - FG \quad BD := \frac{BF}{2}$

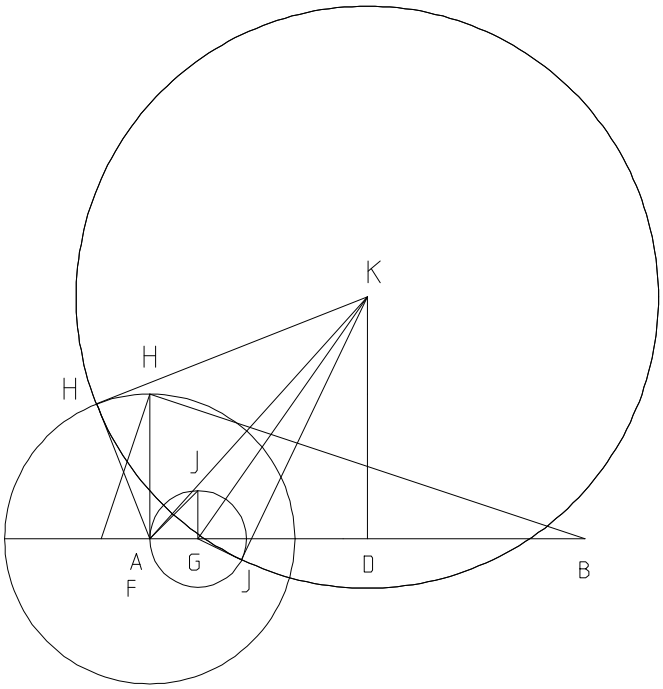
$AD := AB + BD \quad DF := BD$

$DG := DF + FG$

CR "Center to Radius"

$$CR_1 := \frac{1}{2}\cdot\frac{\left(R_1^2 - R_2^2 + D^2\right)}{D} \qquad CR_1 = 4.5 \qquad AD = 4.5$$

$$CR_2 := \frac{1}{2}\cdot\frac{\left(R_2^2 - R_1^2 + D^2\right)}{D} \qquad CR_2 = -3.5 \qquad DG = -3.5$$



Locate length K here $DK := 5$

$$AK := \sqrt{DK^2 + AD^2} \qquad GK := \sqrt{DK^2 + DG^2}$$

$$HK := \sqrt{AK^2 - AH^2} \qquad JK := \sqrt{GK^2 - GJ^2}$$

$HK = 6.021$

LP "Length of Power line"

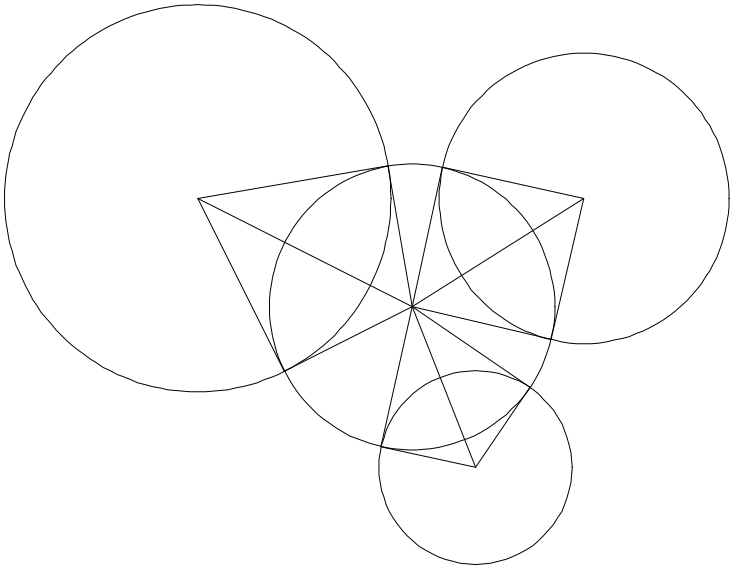
$LP := DK$

LT "Length of radial Tangent" LP is the variable chosen for circle center on the power line.

$$LT := \frac{1}{2}\cdot\left|\frac{\sqrt{4\cdot LP^2\cdot D^2 + R_1^4 - 2\cdot R_1^2\cdot D^2 - 2\cdot R_1^2\cdot R_2^2 + D^4 - 2\cdot D^2\cdot R_2^2 + R_2^4}}{D}\right|$$

$LT = 6.021$ $JK = 6.021$

The process does not seem to recognize any special cases.



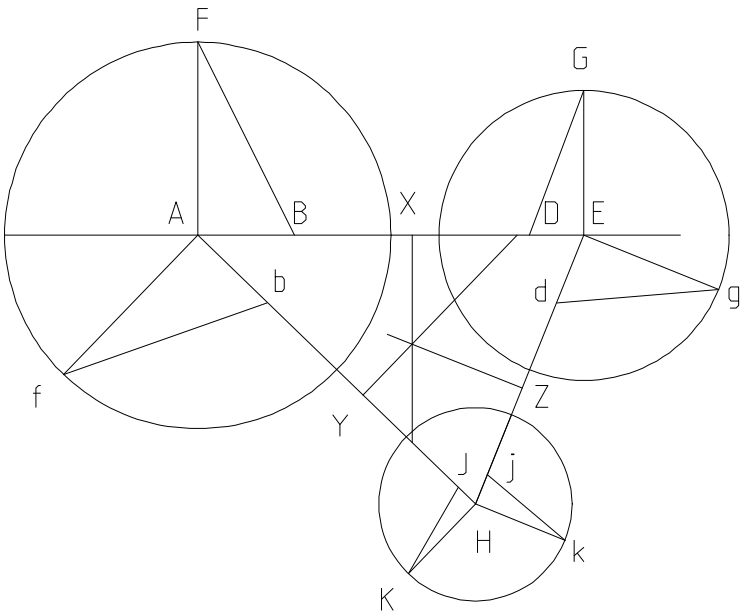
Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the formula for the Power Point and the Length of the resultant Tangent.

The distance between each set of circles is given as D_1, D_2, and D_3. Naturally they must form a triangle.

$D_1 := 4 \quad D_2 := 5 \quad D_3 := 2$

$\Delta := \left(D_1 + D_2 \geq D_3\right) \cdot \left(D_2 + D_3 \geq D_1\right) \cdot \left(D_1 + D_3 \geq D_2\right)$



$\Delta = 1$ Δ "Is this a Triangle?

$R_1 := 3 \quad R_2 := 2 \quad R_3 := 4$

$AE := D_1 \quad AH := D_2 \quad EH := D_3$

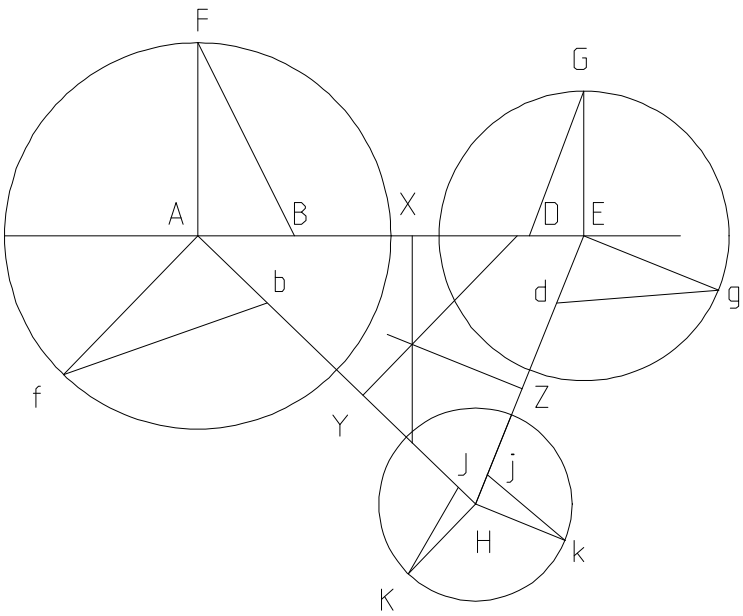
$AF := R_1 \quad HK := R_2 \quad EG := R_3$

$Af := AF \quad Hk := HK \quad Eg := EG$

$AB := \frac{AF^2}{AE} \quad DE := \frac{EG^2}{AE}$

$Ab := \frac{Af^2}{AH} \quad HJ := \frac{HK^2}{AH}$

$Hj := \frac{Hk^2}{EH} \quad Ed := \frac{Eg^2}{EH}$



$BD := AE - AB - DE \quad BX := \frac{BD}{2}$

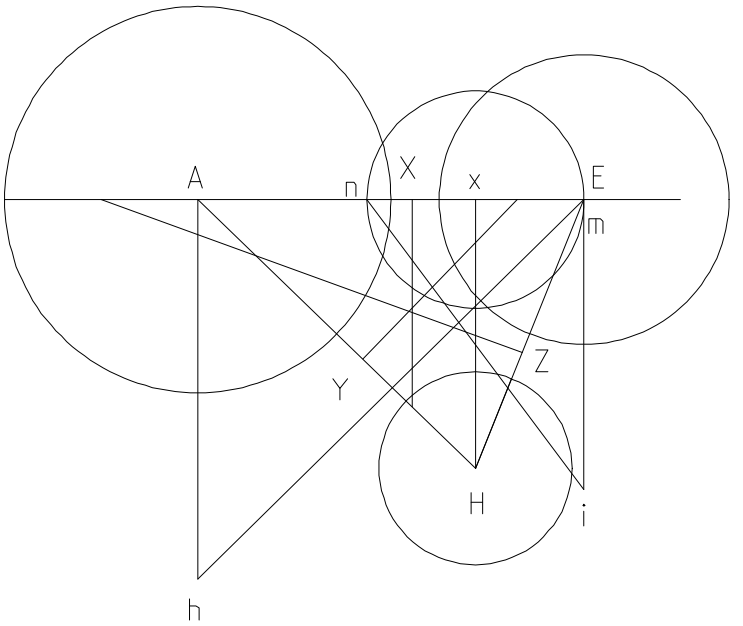
$bJ := AH - Ab - HJ \quad bY := \frac{bJ}{2}$

$dj := EH - Ed - Hj \quad dZ := \frac{dj}{2}$

$AX := AB + BX \quad AX = 1.125$

$AY := Ab + bY$

$EZ := Ed + dZ \quad EZ = 4$



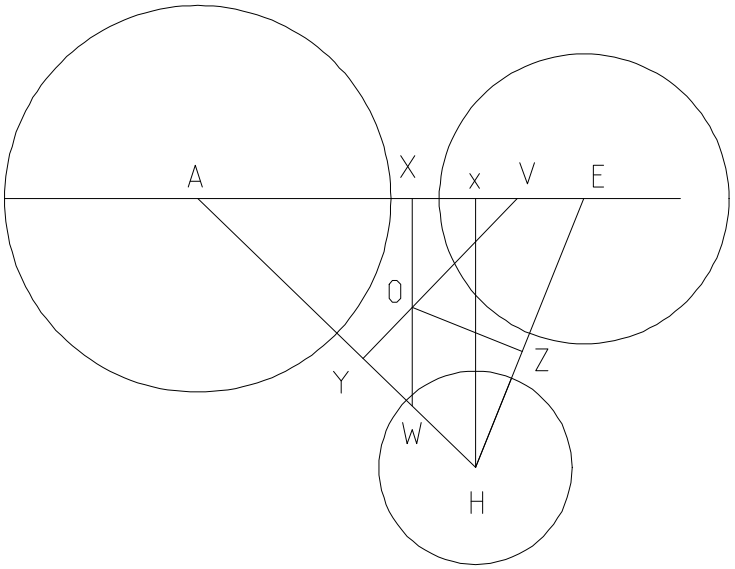
$Ah := AH \quad Ei := EH$

$Am := \frac{Ah^2}{AE} \quad En := \frac{Ei^2}{AE}$

$An := AE - En \quad mn := Am - An$

$nx := \frac{mn}{2} \quad Ax := An + nx$

$Hx := \sqrt{AH^2 - Ax^2}$



$$WX := \frac{Hx \cdot AX}{Ax} \qquad WX = 0.462$$

$$VY := \frac{Hx \cdot AY}{Ax} \qquad VY = 1.232$$

$$AV := \frac{AH \cdot AY}{Ax} \qquad AV = 3.243$$

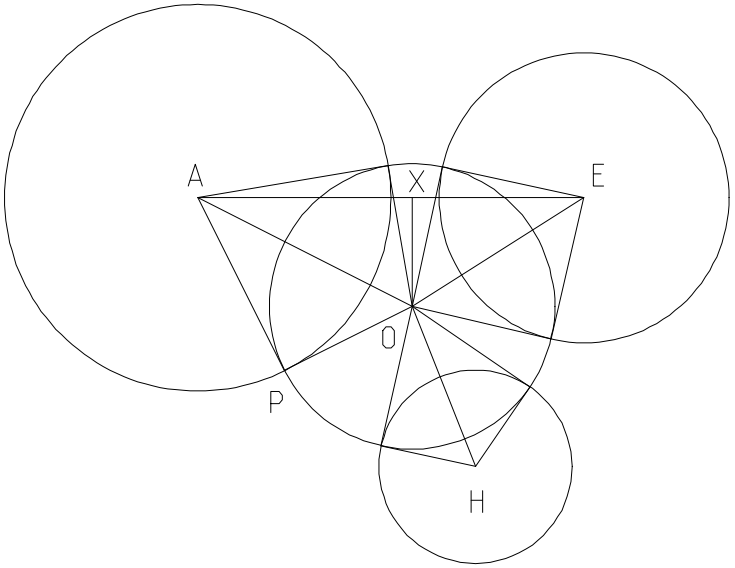
$$VX := AV - AX$$

$$OX := \frac{AY \cdot VX}{VY} \qquad OX = 5.157$$

PP "Power Point"

$$PP := \frac{1}{2} \cdot \frac{\left(\begin{array}{l} D_1^2 \cdot R_1^2 + D_1^2 \cdot D_2^2 - 2 \cdot D_1^2 \cdot R_2^2 + R_1^2 \cdot D_3^2 - R_1^2 \cdot D_2^2 - D_1^4 + D_1^2 \cdot D_3^2 \ldots \\ + R_3^2 \cdot D_1^2 - R_3^2 \cdot D_3^2 + R_3^2 \cdot D_2^2 \end{array} \right)}{\left(D_1 \cdot \sqrt{\left| -2 \cdot D_1^2 \cdot D_2^2 + D_1^4 - 2 \cdot D_1^2 \cdot D_3^2 + D_3^4 - 2 \cdot D_2^2 \cdot D_3^2 + D_2^4 \right|} \right)}$$

$$PP = 5.157$$



$EX := AE - AX$

$AO := \sqrt{AX^2 + OX^2}$ $\quad EO := \sqrt{EX^2 + OX^2}$

$AP := AF$ $\quad EP := EG$

$OP_A := \sqrt{AO^2 - AP^2}$ $\quad OP_E := \sqrt{EO^2 - EP^2}$

$OP_A = 4.342$ $\quad OP_E = 4.342$

LT "Length of Tangent"

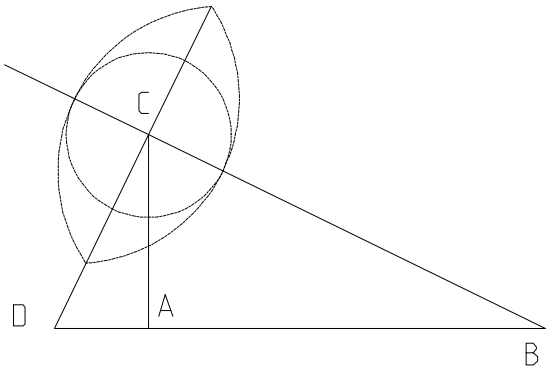
$$LT := \frac{\sqrt{\begin{aligned}&-R_1^4\cdot D_3^2 - R_1^2\cdot R_3^2\cdot D_1^2 - R_1^2\cdot D_3^4 + D_1^2\cdot R_1^2\cdot D_3^2 - D_1^2\cdot D_3^2\cdot D_2^2 \ldots\\&+ R_2^2\cdot R_1^2\cdot D_3^2 - R_2^2\cdot R_1^2\cdot D_2^2 - R_2^2\cdot R_3^2\cdot D_3^2 + R_2^2\cdot R_3^2\cdot D_2^2 \ldots\\&+ R_3^2\cdot D_1^2\cdot D_2^2 - R_3^2\cdot D_2^4 - R_3^4\cdot D_2^2 + D_1^2\cdot R_1^2\cdot R_2^2 + D_1^2\cdot D_2^2\cdot R_2^2 \ldots\\&+ D_1^2\cdot R_2^2\cdot D_3^2 + D_1^2\cdot R_2^2\cdot R_3^2 + R_1^2\cdot R_3^2\cdot D_3^2 + R_3^2\cdot D_3^2\cdot D_2^2 - D_1^2\cdot R_2^4 \ldots\\&+ -D_1^4\cdot R_2^2 + R_1^2\cdot D_3^2\cdot D_2^2 + R_1^2\cdot R_3^2\cdot D_2^2\end{aligned}}}{\sqrt{-2\cdot D_1^2\cdot D_2^2 + D_1^4 - 2\cdot D_1^2\cdot D_3^2 + D_3^4 - 2\cdot D_3^2\cdot D_2^2 + D_2^4}}$$

$LT = 4.342$

Division, A^2

04_30_94.MCD

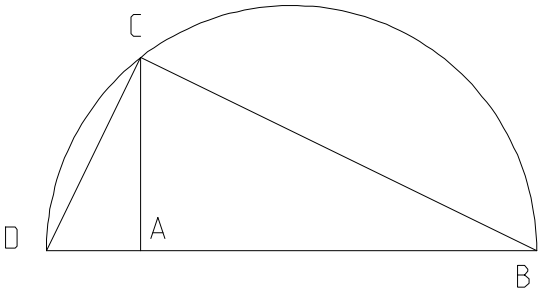
One does not work with geometry often, so it may be that one does not keep basics in mind when trying to work a figure. This little paper is about a basic move. I bring this to light, as I have seen a ratio often given as division. In geometry, so far as I know, one cannot divide a line by a line, but can form a series of the nature $A^N : B^N$ as a linear figure. Given A and B one can raise them to any whole power simultaneously with a couple of simple moves based on the figure immediately below. See work done in 1995.

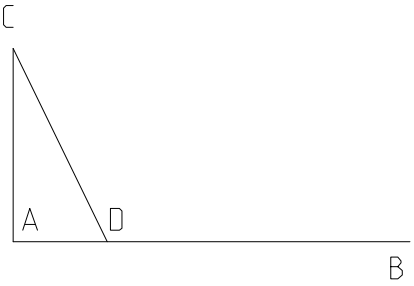


Divide AC^2 by AB.

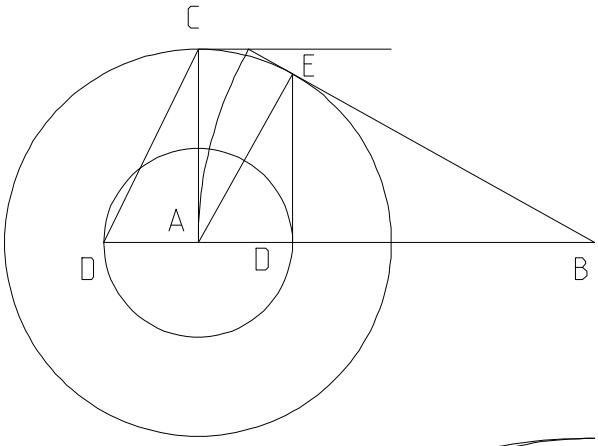
Process Summary

I have noticed that my solutions depend upon this basic move.

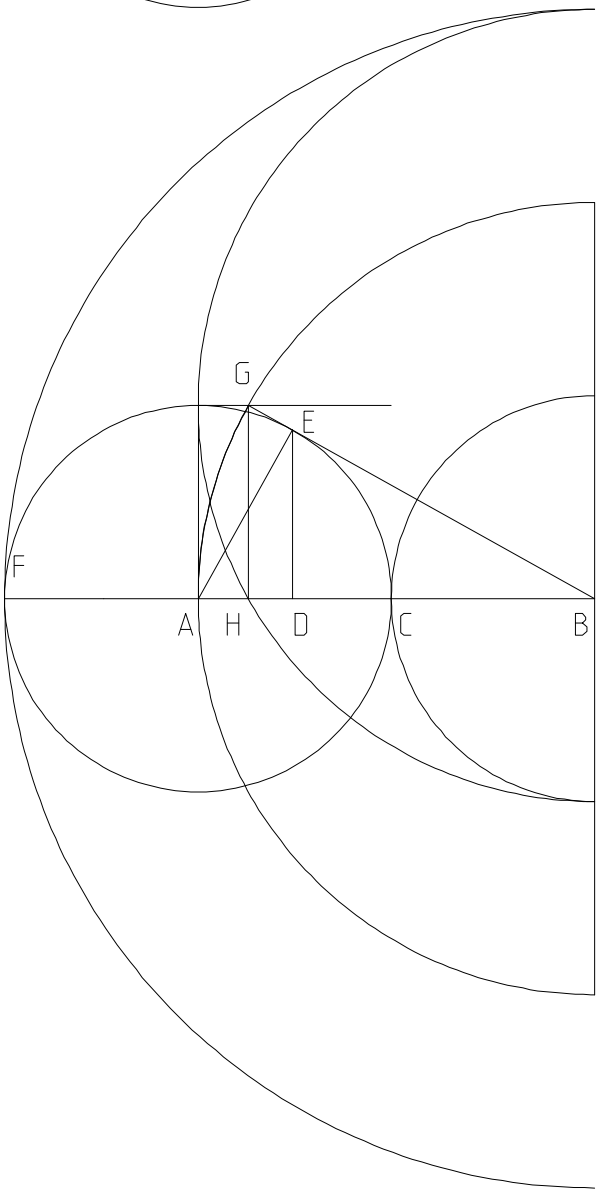




In some works I have represent it simply as the figure on the left. One may realize that in my Pythagorean Completion I used the circular form and it is often expressed as a pole and polar arrangement. A terminology that does not seem fit for the processes that they represent, physical and not mathematical.



The jargon is that B is called a Pole and D is on a polar, a segment of which is DE. But it can easily be seen that the figure is a transformation. It is another way of dividing AC^2 by AB. The figure now raises a question for me. I had thought that I answered it previously, but I cannot find it in my files. Is BE always collinear with BG? This paper is helping me mediate on poles and polars, which names do not help me understand the true ratio involved. Mathematicians seem to like a proliferation of names. And, god forbid, B and D are called conjugate in respect to each other. I have a hard enough time remembering my own name, that is why I keep my id. (in the Freudian sense) close at hand.



$AC := 5 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CF := 2 \cdot AC \quad AG := AC \quad BF_\delta := CF + BC_\delta$

$BH_\delta := \sqrt{BF_\delta \cdot BC_\delta} \quad GH := AC$

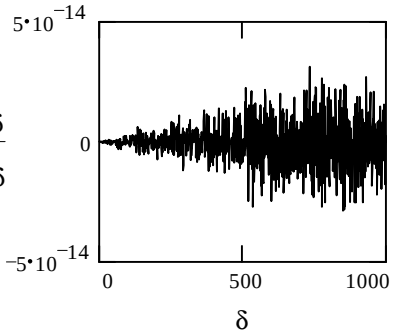
$BG_\delta := \sqrt{(BH_\delta)^2 + GH^2} \quad AE := AC$

$AB_\delta := AC + BC_\delta \quad AD_\delta := \frac{AE^2}{AB_\delta}$

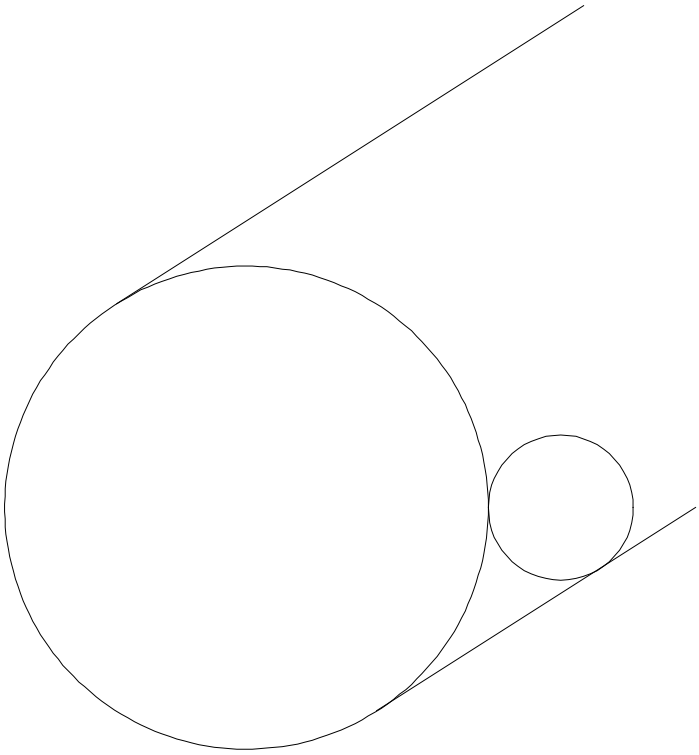
$BE_\delta := \sqrt{(AB_\delta)^2 - AE^2} \quad BD_\delta := AB_\delta - AD_\delta$

$DE_\delta := \sqrt{(BE_\delta)^2 - (BD_\delta)^2}$

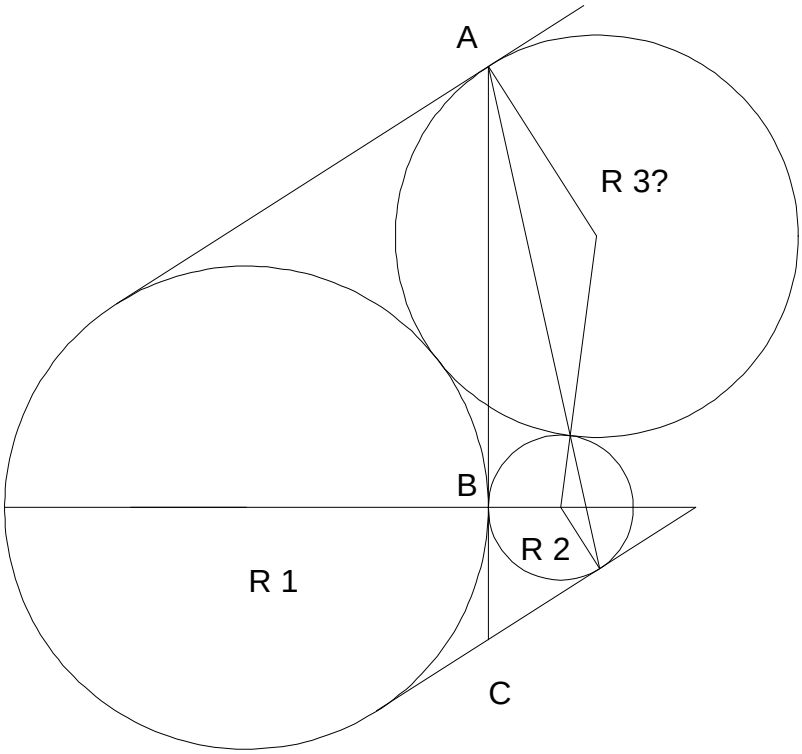
$\frac{GH}{BH_\delta} - \frac{DE_\delta}{BD_\delta}$



Two Circles and a Parallel

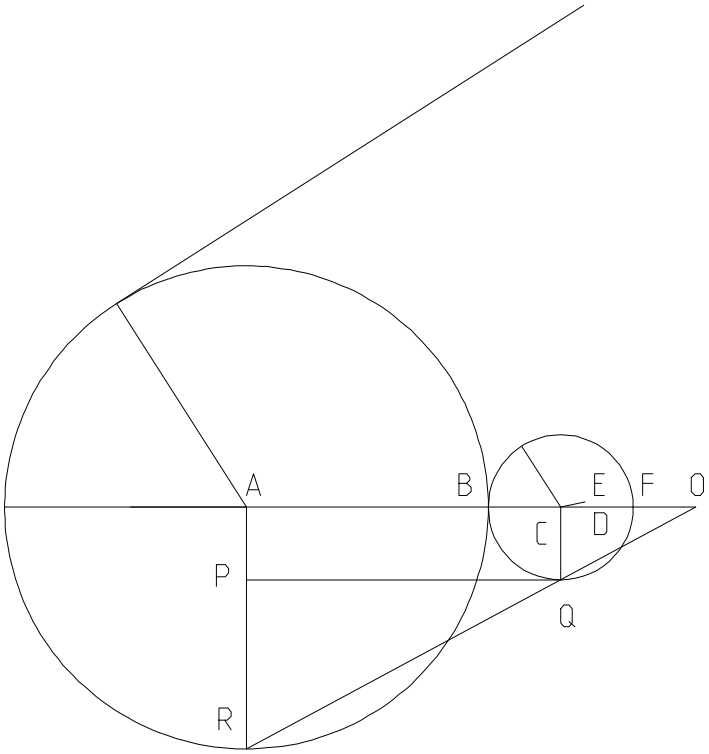


What I would like to do is to is drop in a circle that is tangent to the given two circles that are already tangent and tangent to the line from the similarity point and also have this circle tangent to the parallel of the similarity line that lies tangent to the first circle. The formula derived for my process tells me that I will do it the hard way. It will ***predict*** an easier method.



Process Summary

Given the radius of the two circles, what is the radius of the third? Attempt to develop a formula for the resultant radius. And also, of the power line between parallels, what is the ratio of AC:BC in terms of the given radius' ?



Find the Similarity Point.

$AB := R_1 \quad CF := R_2 \quad BC := CF$

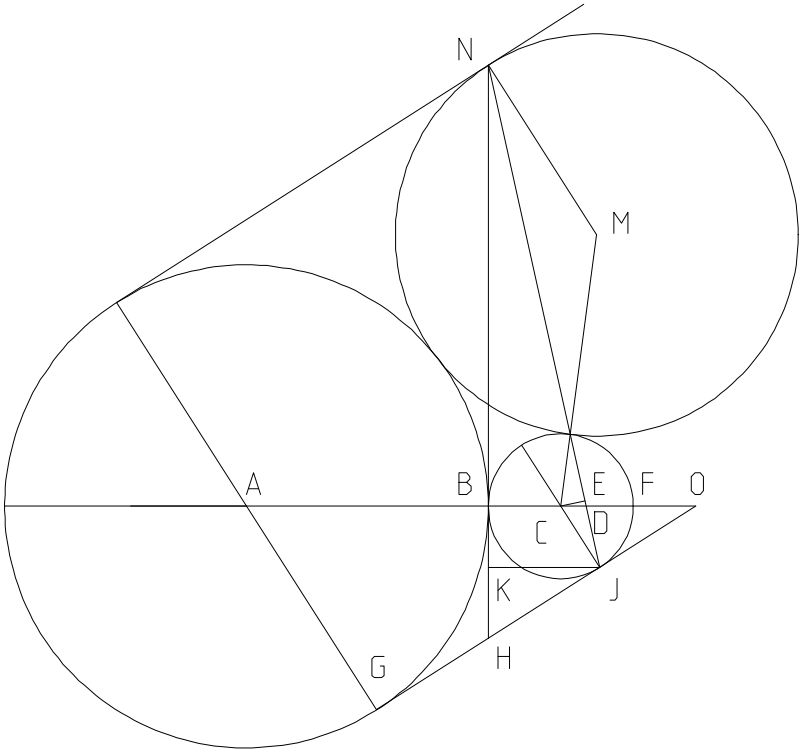
$CB := CF \quad AC := AB + BC$

$PQ := AC \quad AR := AB$

$CQ := CF \quad AP := CQ$

$PR := AR - AP$

$AO := \frac{PQ \cdot AR}{PR}$



Find the segment of the power line (HN) between parallels.

$AG := AB \quad GO := \sqrt{AO^2 - AG^2}$

$BO := AO - AB$

$BH := \frac{AG \cdot BO}{GO} \quad BN := \frac{BH \cdot AB}{BC}$

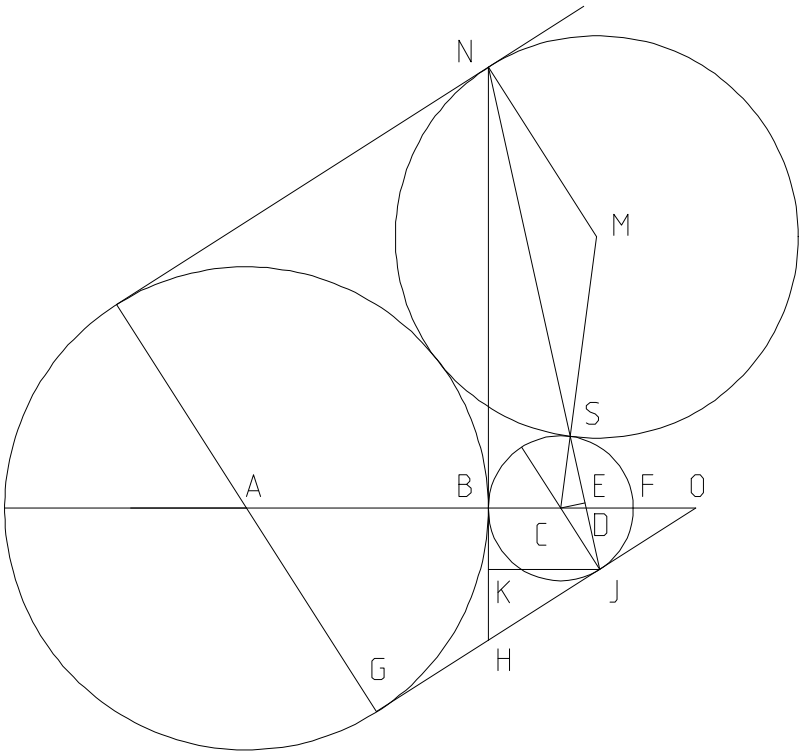
$HO := \frac{AO \cdot BH}{AG} \quad CO := BO - BC$

$JO := \frac{GO \cdot CO}{AO} \quad HN := BH + BN$

Find JN

$HJ := HO - JO \quad KH := \frac{AG \cdot HJ}{AO}$

$KN := HN - KH$



$KJ := \sqrt{HJ^2 - KH^2}$

$JN := \sqrt{KN^2 + \text{Find MS}}$

$BD := \frac{KJ \cdot BN}{KN}$ $CD := BD - BC$

$CE := \frac{KN \cdot CD}{JN}$ $CJ := BC$

$EJ := \sqrt{CJ^2 - CE^2}$ $CS := 2 \cdot EJ$

$NS := JN - JS$ $CS := BC$

$MS := \frac{CS \cdot NS}{JS}$

Plug Values in Here

$R_1 \equiv 8$ $R_2 \equiv 6$

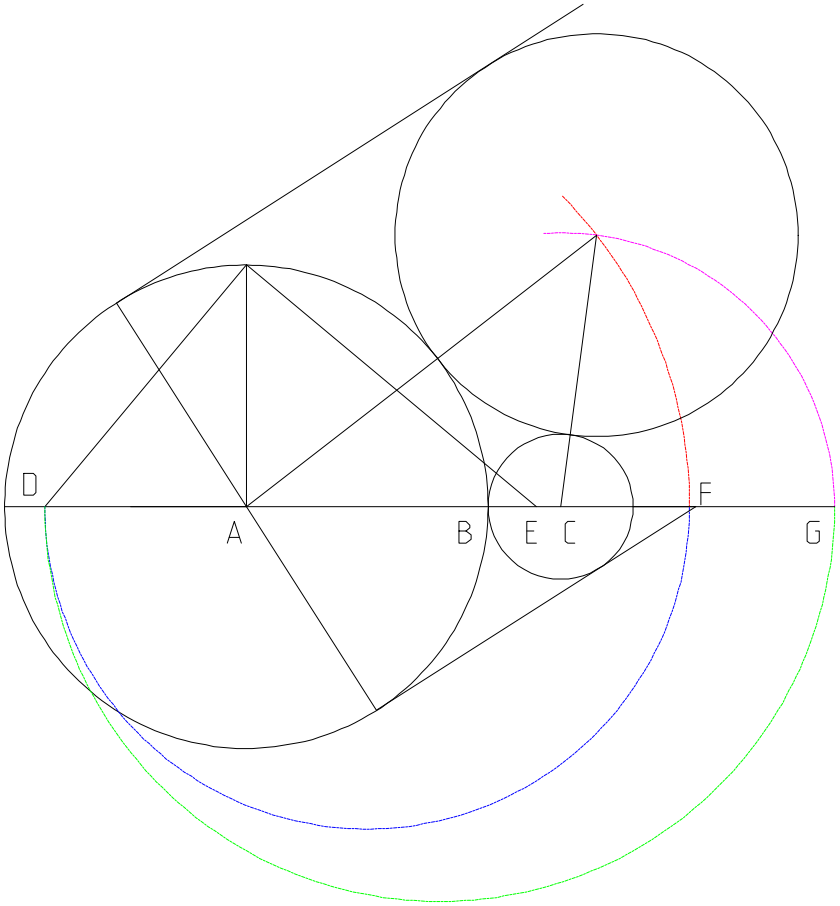
There was too much work here for the symbolic processor to reduce all the equations to one, easily. It took me three days to nurse the processor through it, and this is a short work. The formula does not have the "sour" spot. (Place both R_1 and R_2 to the same value to see it.) When R_2 is 1/4 of R_1, there are six tangents.

$$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad R_3 = 2.667 \qquad MS = 2.667$$

The formula tells me at least two things, 1) There is a second method to solve the problem, 2) the process is a rather baroque method of dividing a square.

$$\frac{HN}{BN} = 1.75 \qquad \frac{R_1 + R_2}{R_1} = 1.75 \qquad \frac{HN}{BH} = 2.333 \qquad \frac{R_1 + R_2}{R_2} = 2.333$$

The results for the ratios of the power line segment are very nice also.



What is the construction suggested by the found formula of

$$\frac{R_1^2}{4 \cdot R_2} \quad ?$$

$AB := R_1 \quad BC := R_2$

$AE := 4 \cdot BC$

$AD := \frac{AB^2}{AE}$

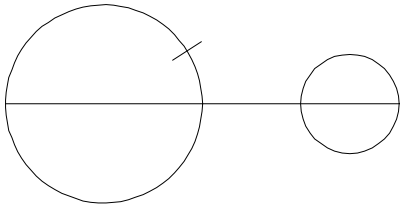
$AF := AB + AD$

$CG := BC + AD$

$R_3 = 2.667 \qquad AD = 2.667$

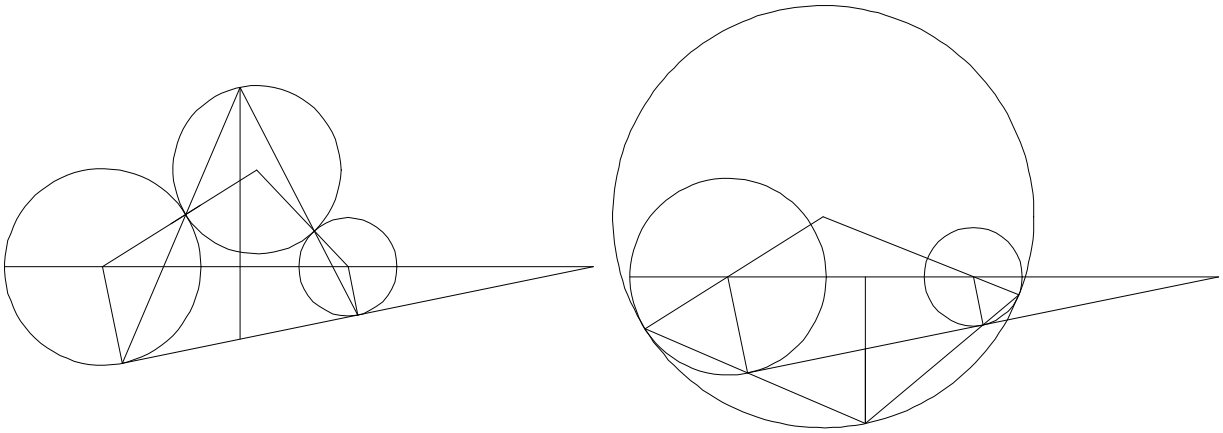
For those who may become confused as to the dashed lines, the segment AD is added to the radius of the both circles their intersection is the center of the circle sought.

Two Circles, given a tangent on one.

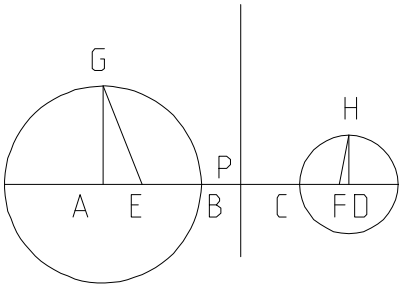


Given two circles and a point that is on the circumference of one, find a circle tangent to the circle at that point and also tangent to the other circle. The convention for this point will be "from the power line".

Process Summary



Find the power line.



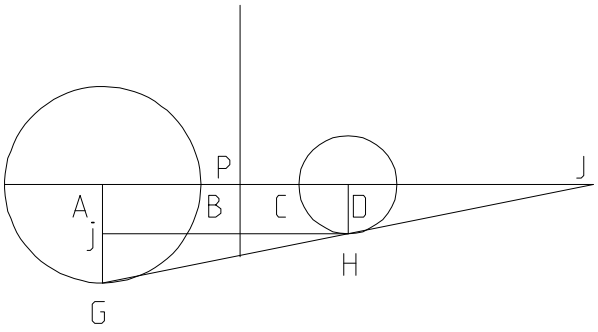
$AB := R_1 \quad CD := R_2 \quad AD := D$

$AG := AB \quad DH := CD \quad AE := \frac{AG^2}{AD}$

$DF := \frac{DH^2}{AD} \quad EF := AD - AE - DF$

$EP := \frac{EF}{2} \quad AP := AE + EP \quad DP := DF + EP$

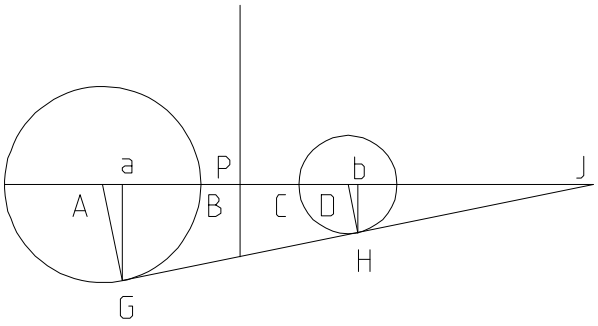
$BP := AP - AB$



$Hj := AD \quad Aj := DH$

$Gj := AG - DH \quad AJ := \frac{Hj \cdot AG}{Gj}$

$DJ := AJ - AD \quad HJ := \sqrt{DJ^2 - DH^2}$



$GJ := \sqrt{AJ^2 - AG^2} \quad Aa := \frac{AG^2}{AJ}$

$Ga := \sqrt{AG^2 - Aa^2} \quad Db := \frac{DH^2}{DJ}$

$Hb := \sqrt{DH^2 - Db^2}$

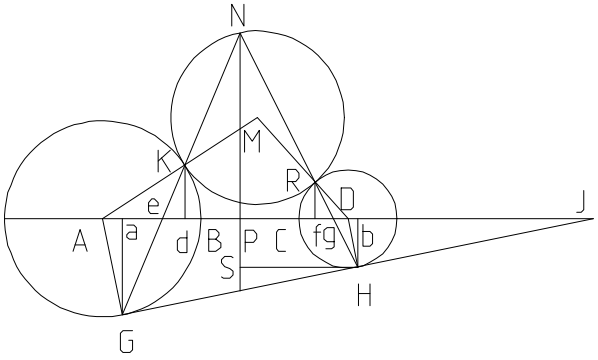
$Ba := AB - Aa$

$Pb := DP + Db$

$P := \left| \text{if}(P \le 2 \cdot AB, P, 0) \right|$

$Bd := P$

$AK := AB \quad ad := Ba - Bd$



$Ad := AB - Bd \quad Kd := \sqrt{AK^2 - Ad^2}$

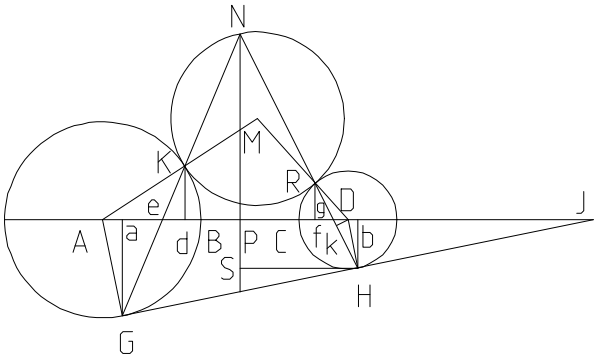
$de := \frac{ad \cdot Kd}{Kd + Ga} \quad Pe := BP + Bd + de$

$NP := \frac{Kd \cdot Pe}{de} \quad HS := Pb \quad PS := Hb$

$NS := NP + PS \quad Pg := \frac{HS \cdot NP}{NS}$

$Dg := DP - Pg \quad bg := Dg + Db$

$Hg := \sqrt{bg^2 + Hb^2}$



To save clutter, see 07_18_93.MCD Mod C.

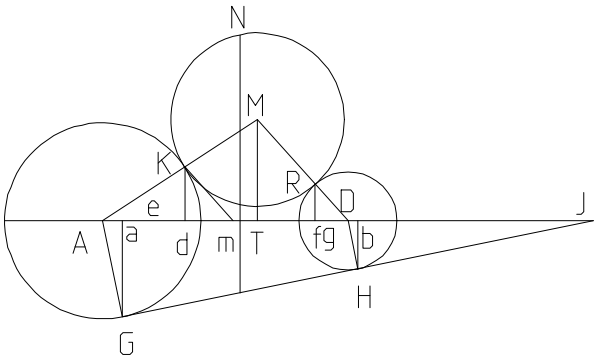
$$gk := \frac{1}{2}\cdot\frac{Dg^2}{Hg} + \frac{1}{2}\cdot Hg - \frac{1}{2}\cdot\frac{DH^2}{Hg}$$

$Dk := \sqrt{Dg^2 - gk^2} \quad Hk := \sqrt{DH^2 - Dk^2}$

$Rg := Hk - gk$

$$Df := \frac{1}{2}\cdot\frac{CD^2}{Dg} + \frac{1}{2}\cdot Dg - \frac{1}{2}\cdot\frac{Rg^2}{Dg}$$

$DR := CD \quad Rf := \sqrt{DR^2 - Df^2}$



$$dm := \frac{Df\cdot Kd}{Rf} \quad Am := Ad + dm$$

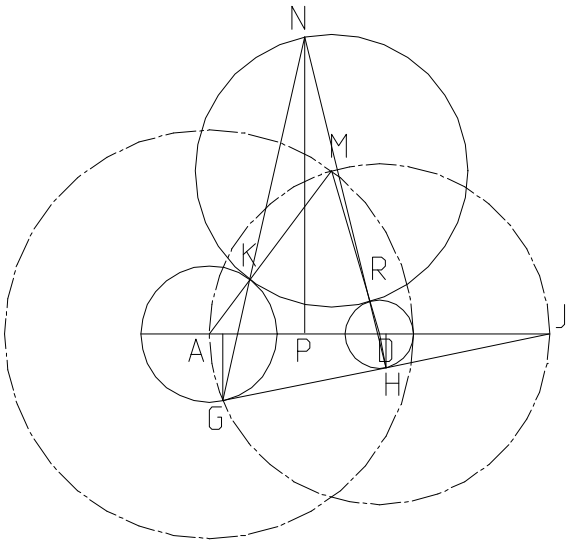
$$MT := \frac{Kd\cdot AD}{Am} \quad AM := \frac{AK\cdot MT}{Kd}$$

$KM := AM - AK$

$R_3 := |KM| \quad P = 19$

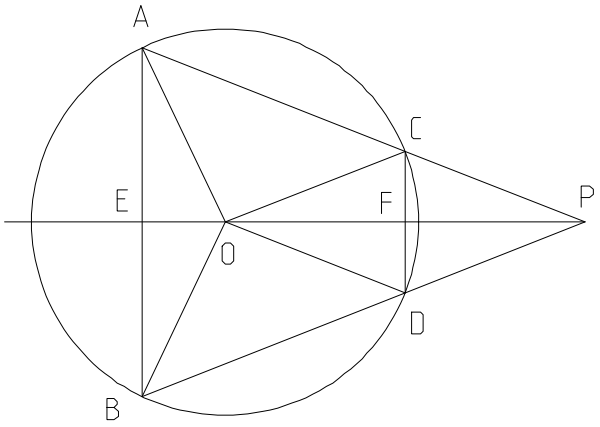
Plug Values in Here. Results is in P_3.

$R_1 \equiv 10 \; R_2 \equiv 3 \quad D \equiv 16 \quad P \equiv 19 \quad R_3 = 14.836$



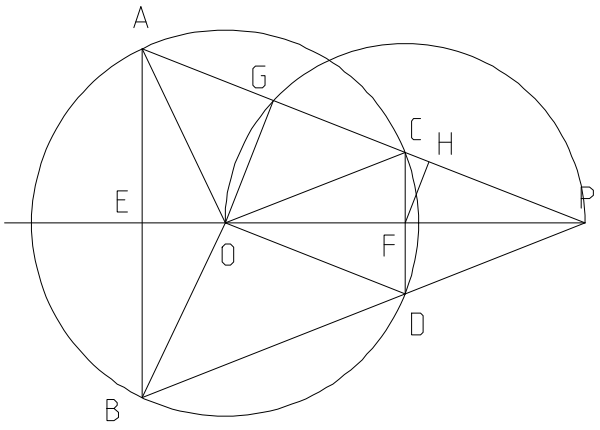
Let us say that instead of choosing a point of tangency upon one of the circles, I wish to place a given circle tangent to both. Derive the name of the length of the power line indicated and find th points of tangency.

Work in progress.



Given that CP = CO what is the relationship between AE and CF?

In order to derive a resonable answer, I have constrained the figure to a maximum of 180º. You will recall that the figure is capable of 270º.



$$CF_\delta := \frac{\frac{1}{2}\cdot\sqrt{2}\cdot CP}{\delta} \qquad FP_\delta := \sqrt{CP^2 - \left(CF_\delta\right)^2}$$

$$CH_\delta := \frac{\left(CF_\delta\right)^2}{CP} \qquad FH_\delta := \sqrt{\left(CF_\delta\right)^2 - \left(CH_\delta\right)^2}$$

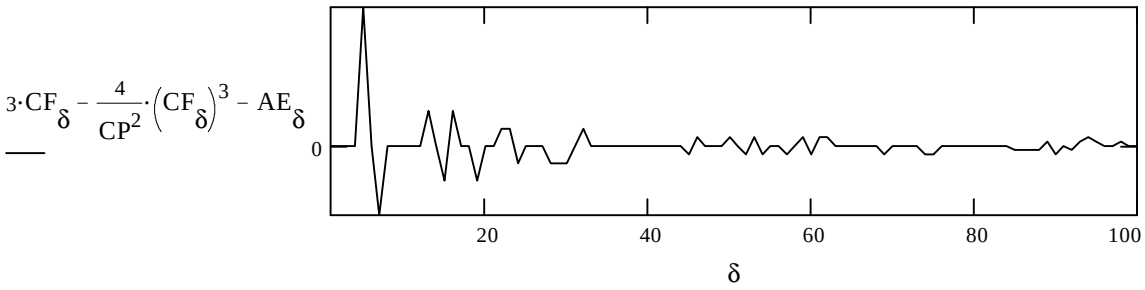
$$OP_\delta := 2\cdot FP_\delta \qquad GO_\delta := \frac{FH_\delta\cdot OP_\delta}{FP_\delta} \qquad CO := CP$$

$$CG_\delta := \sqrt{CO^2 - \left(GO_\delta\right)^2} \qquad AC_\delta := 2\cdot CG_\delta$$

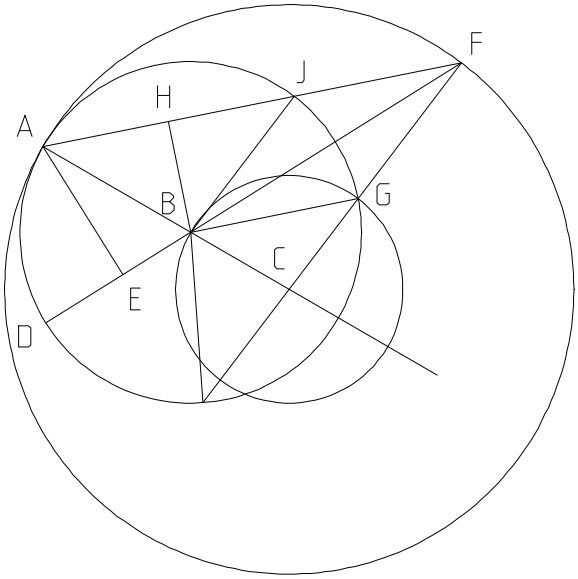
$$AP_\delta := AC_\delta + CP \qquad AE_\delta := \frac{CF_\delta\cdot AP_\delta}{CP}$$

CF is adjusted through D and cannot be less than 1 for the ratio to hold. This constrains the answer to between 0º and 180º.

$$CP \equiv 10 \qquad \delta \equiv 1 \,..\, 100 \qquad 3\cdot CF - \frac{4}{CP^2}\cdot CF^3 - AE = 0$$



The resultant equation seems to support my earlier statement that the same tool used on the cube root figure could also be used on the trisector. To eliminate the radius, simply set it equal to 1.



What is the relationship of DE to BC? $D := 4$

$AB := 99 \quad BC := \frac{AB}{2}\cdot D \quad BG := AB$

$AC := AB + BC \quad AF := \frac{BG\cdot AC}{BC} \quad FJ := AB$

$AJ := AF - FJ \quad HJ := \frac{AJ}{2} \quad BJ := AB$

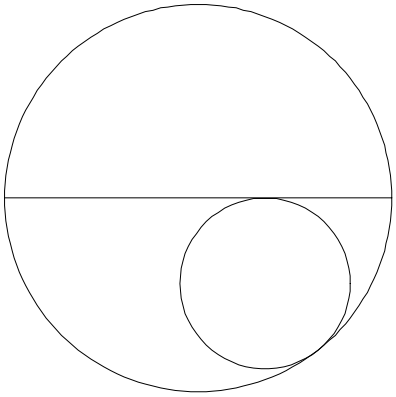
$BH := \sqrt{BJ^2 - HJ^2} \quad FH := HJ + FJ$

$BF := \sqrt{BH^2 + FH^2} \quad BD := AB \quad DF := BF + BD$

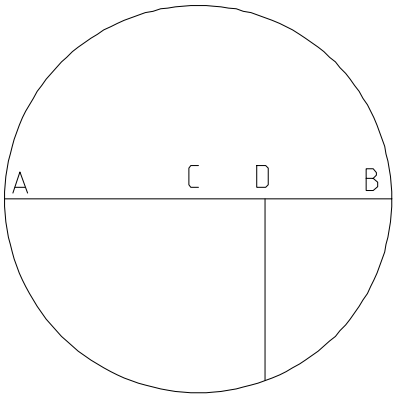
$EF := \frac{FH\cdot AF}{BF} \quad DE := DF - EF \quad DE = 138.133$

$$\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}} + AB - \left[\frac{1}{2}\cdot AB\cdot\frac{(AB + BC)}{BC} + \frac{1}{2}\cdot AB\right]\cdot AB\cdot\frac{(AB + BC)}{\left[BC\cdot\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}}\right]} = 138.133$$

$$\frac{-1}{2}\cdot AB\cdot\frac{\left[-\sqrt{2\cdot BC + AB}\cdot BC - 2\cdot BC^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{2\cdot BC + AB}\right]}{BC^{\left(\frac{3}{2}\right)}} = 138.133$$



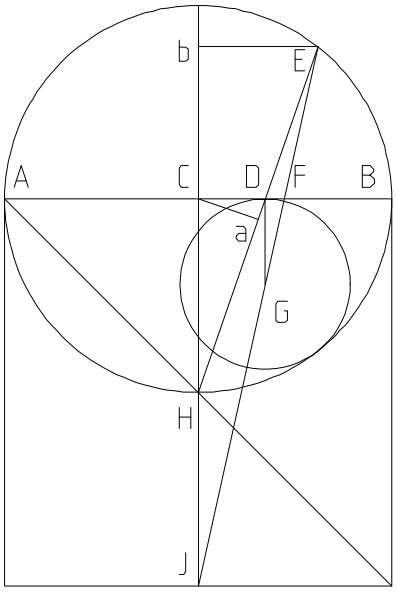
The smaller circle is tangent to the diameter of the larger and also tangent to circumference of the larger. Given the point of tangency on the diameter, what is the radius that will make it tangent to the circumference?



Given a point on the diameter, what is the radius of the inner tangent circle?

$$AB := 100 \quad AC := \frac{AB}{2} \quad CB := AC$$

$$\Delta := 100 \quad \delta := 1 .. \Delta \quad CD_\delta := \frac{CB}{\Delta} \cdot \delta$$



$$CH := AC \quad DH_\delta := \sqrt{\left(CD_\delta\right)^2 + CH^2}$$

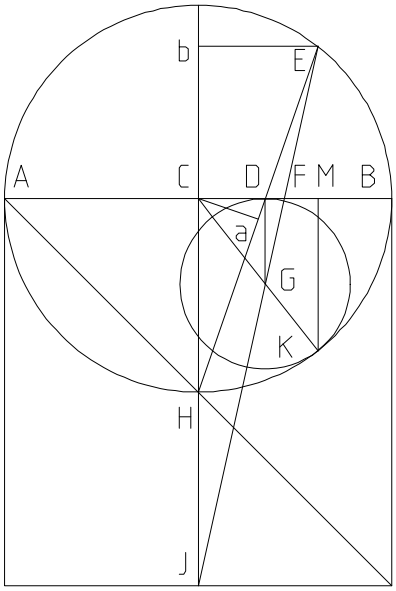
$$Ha_\delta := \frac{CH^2}{DH_\delta} \quad EH_\delta := 2 \cdot Ha_\delta \quad HJ := AC$$

$$Eb_\delta := \frac{CD_\delta \cdot EH_\delta}{DH_\delta} \quad Hb_\delta := \sqrt{\left(EH_\delta\right)^2 - \left(Eb_\delta\right)^2}$$

$$Cb_\delta := Hb_\delta - CH \quad CJ := CH + HJ$$

$$Jb_\delta := CJ + Cb_\delta \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

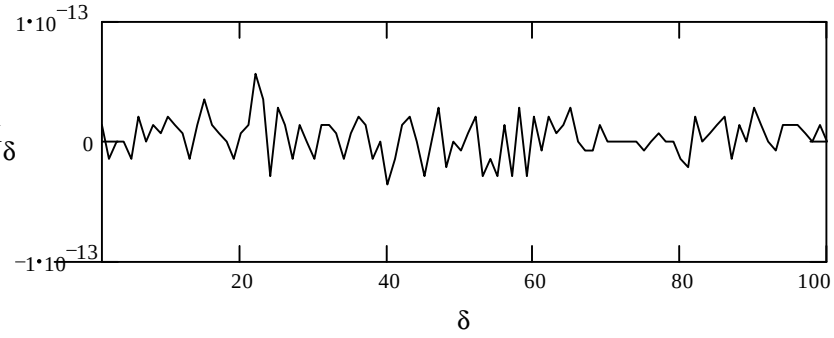


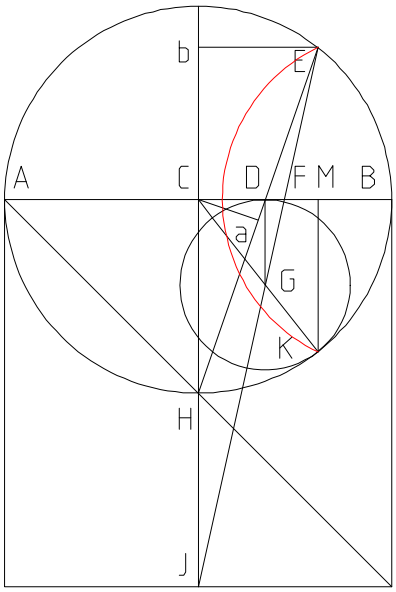
Does CG + GK = AC ?

$CG_\delta := \sqrt{(CD_\delta)^2 + (DG_\delta)^2}$ $GK_\delta := DG_\delta$

$CK_\delta := CG_\delta + GK_\delta$ $\sum_\delta (AC - CK_\delta) = 3.908 \cdot 10^{-13}$

$AC - CK_\delta$

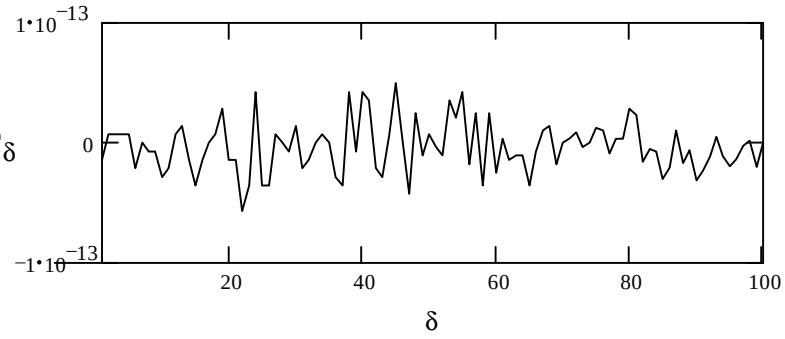


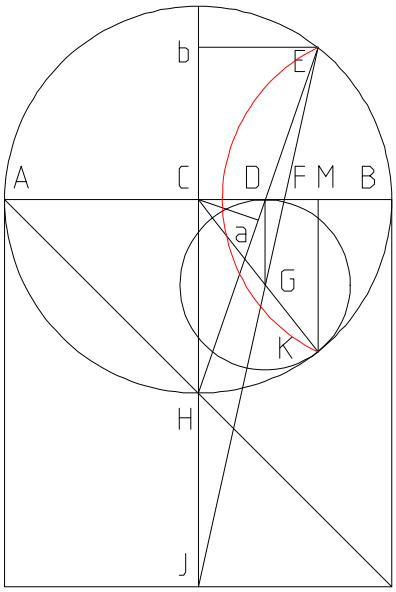


Does Cb = KM?

$KM_\delta := \frac{DG_\delta \cdot CK_\delta}{CG_\delta}$ $\sum_\delta (KM_\delta - Cb_\delta) = -3.524 \cdot 10^{-13}$

$KM_\delta - Cb_\delta$

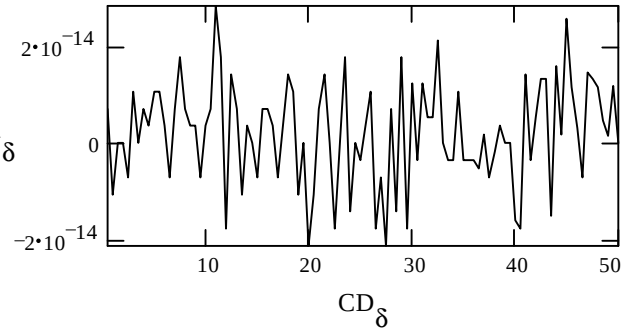


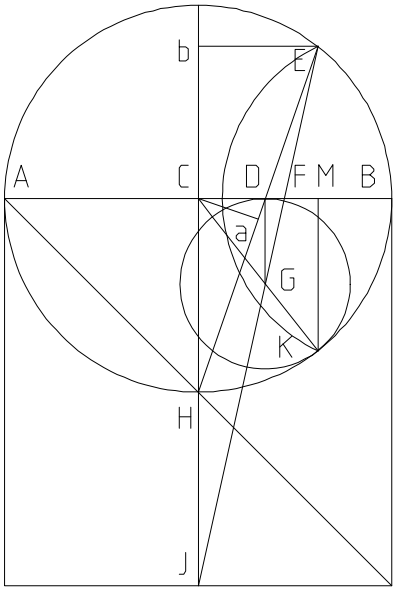


What is the formula for DG when given CD?

$$\sum_{\delta}\left[\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}\right] = 2.072\cdot 10^{-13}$$

$$\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}$$





Given the point on the radius, find DG.

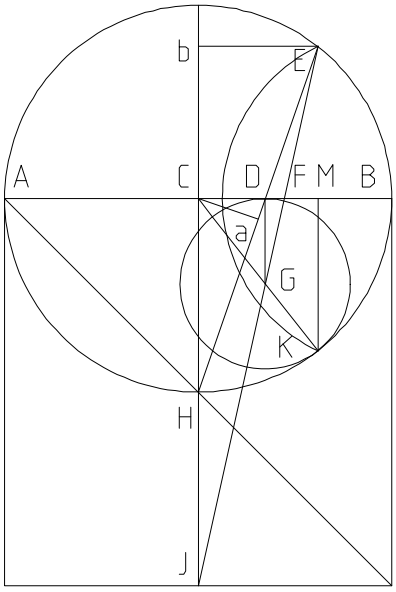
$AB := 4 \quad AC := \frac{AB}{2} \quad CB := AC \quad CM_{\delta} := \frac{CB}{\Delta}\cdot\delta$

$AM_{\delta} := AC + CM_{\delta} \quad BM_{\delta} := CB - CM_{\delta}$

$KM_{\delta} := \sqrt{AM_{\delta}\cdot BM_{\delta}} \quad Cb_{\delta} := KM_{\delta}$

$CH := AC \quad HJ := AC \quad CJ := CH + HJ$

$Jb_{\delta} := CJ + Cb_{\delta} \quad Hb_{\delta} := CH + Cb_{\delta}$



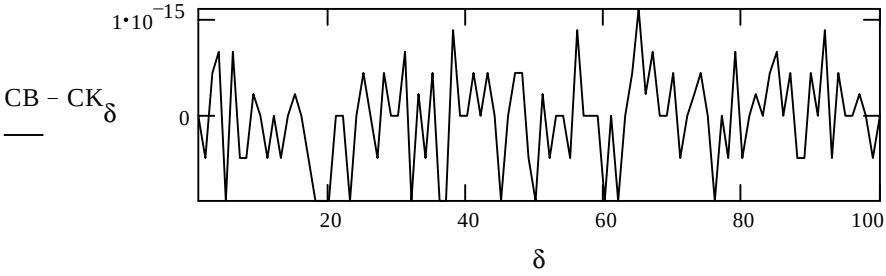
$$Eb_\delta := CM_\delta \quad CD_\delta := \frac{Eb_\delta \cdot CH}{Hb_\delta} \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

Does CK = CB? Make sure there is no typo.

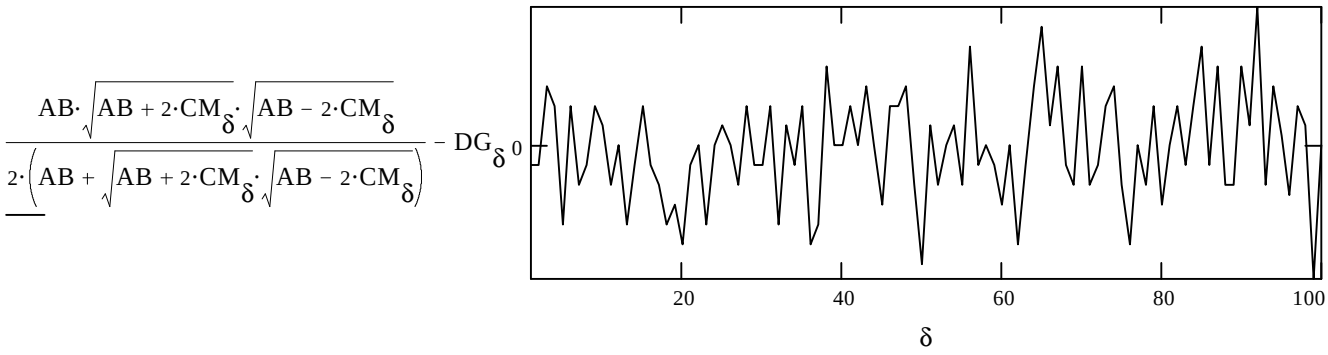
$$CG_\delta := \sqrt{\left(CD_\delta\right)^2 + \left(DG_\delta\right)^2} \quad GK_\delta := DG_\delta$$

$$CK_\delta := CG_\delta + GK_\delta \quad \sum_\delta \left(CB - CK_\delta\right) = -3.331 \cdot 10^{-15}$$

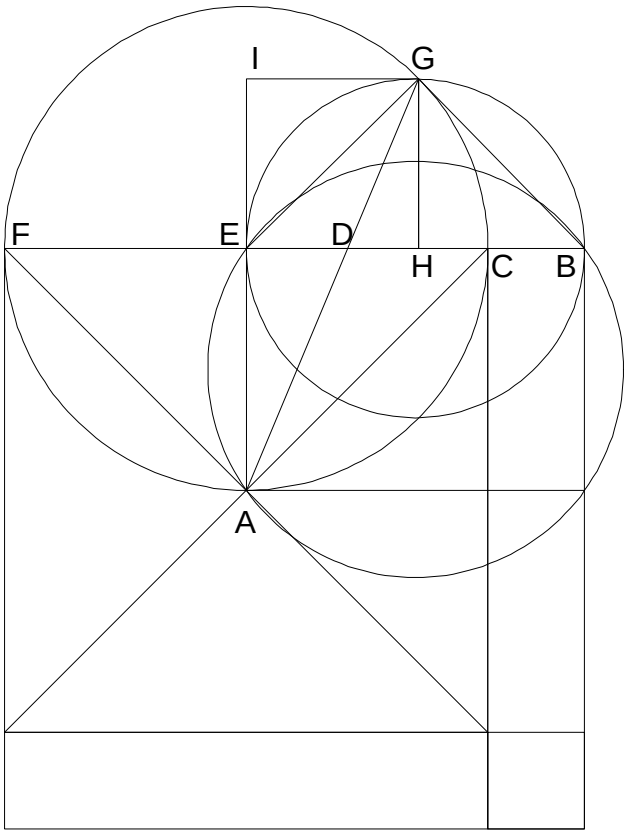


What is the formula for DG, given CM, the perpendicular to the point on the circumference?

$$\sum_\delta \left[\frac{AB \cdot \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta}}{2 \cdot \left(AB + \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta} \right)} - DG_\delta \right] = -1.749 \cdot 10^{-15}$$



Trivial method for doing Square Roots.



If I draw a circle BE, then a line AG, BD is the square root of $BC \times BF$.

$CF := 1000 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CE := \frac{CF}{2} \quad BE_\delta := BC_\delta + CE$

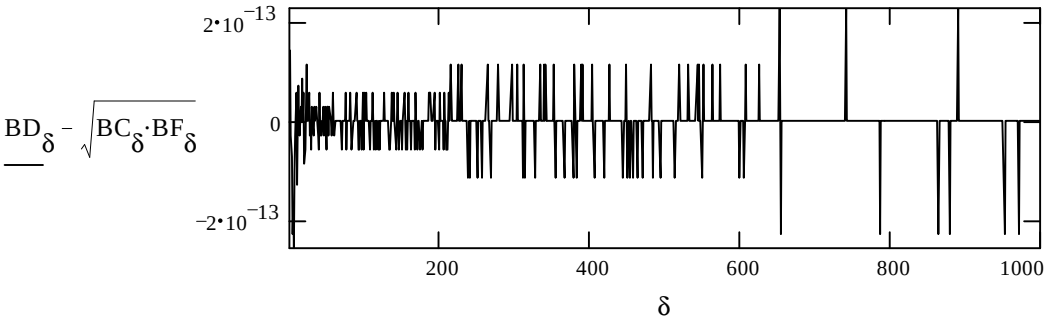
$EG := CE \quad BG_\delta := \sqrt{(BE_\delta)^2 - EG^2}$

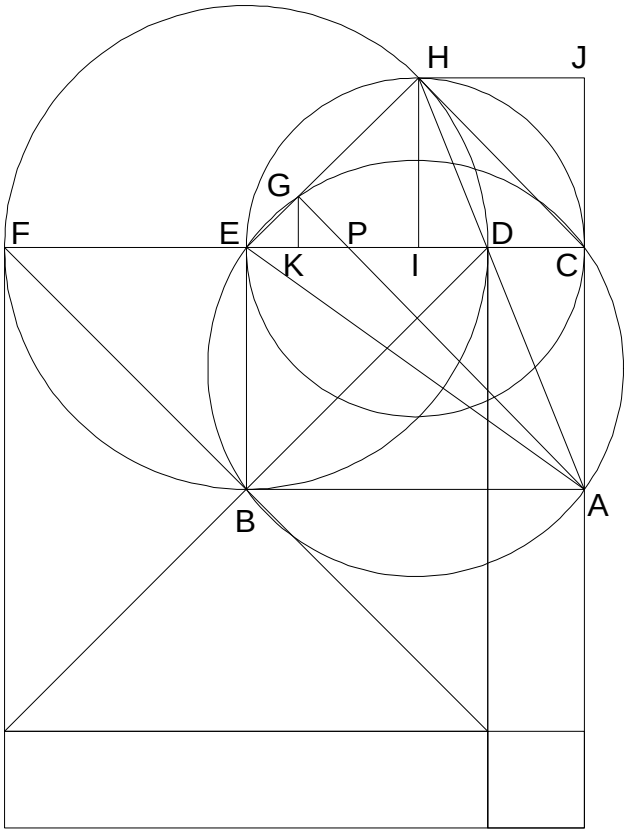
$EH_\delta := \frac{EG^2}{BE_\delta} \quad GH_\delta := \sqrt{EG^2 - (EH_\delta)^2}$

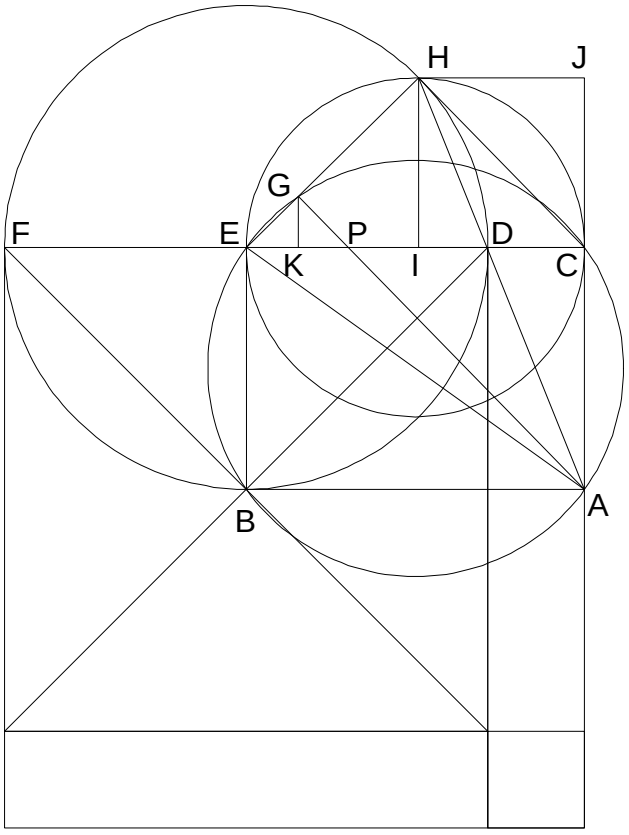
$GI_\delta := EH_\delta \quad EI_\delta := GH_\delta \quad AE := CE$

$AI_\delta := AE + EI_\delta \quad DE_\delta := \frac{GI_\delta \cdot AE}{AI_\delta}$

$BD_\delta := BE_\delta - DE_\delta \quad BF_\delta := BC_\delta + CF$







10_28_94.MCD

Trivial method for doing Square Root.

If I drop in a circle CHE, another ABE, and a right triangle, CHE, then AG provides CP which is the square root of $CD \times CF$.

$DF := 10 \quad \delta := 1 .. 100 \quad CD_\delta := \delta$

$DE := \frac{DF}{2} \quad AC := DE \quad CE_\delta := DE + CD_\delta$

$EH := DE \quad AE_\delta := \sqrt{AC^2 + (CE_\delta)^2}$

$CH_\delta := \sqrt{(CE_\delta)^2 - EH^2} \quad EI_\delta := \frac{EH^2}{CE_\delta}$

$HI_\delta := \sqrt{EH^2 - (EI_\delta)^2} \quad CJ_\delta := HI_\delta$

$CI_\delta := CE_\delta - EI_\delta \quad HJ_\delta := CI_\delta$

$AJ_\delta := AC + CJ_\delta \quad AH_\delta := \sqrt{(AJ_\delta)^2 + (HJ_\delta)^2}$

$HG_\delta := \frac{1}{2} \cdot \frac{(AH_\delta)^2}{EH} + \frac{1}{2} \cdot EH - \frac{1}{2} \cdot \frac{(AE_\delta)^2}{EH}$

$EG_\delta := EH - HG_\delta$

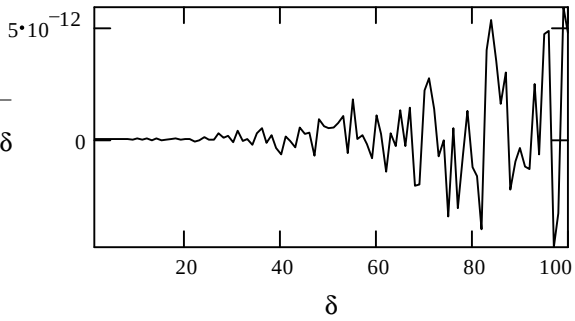
$AG_\delta := \sqrt{(AE_\delta)^2 - (EG_\delta)^2}$

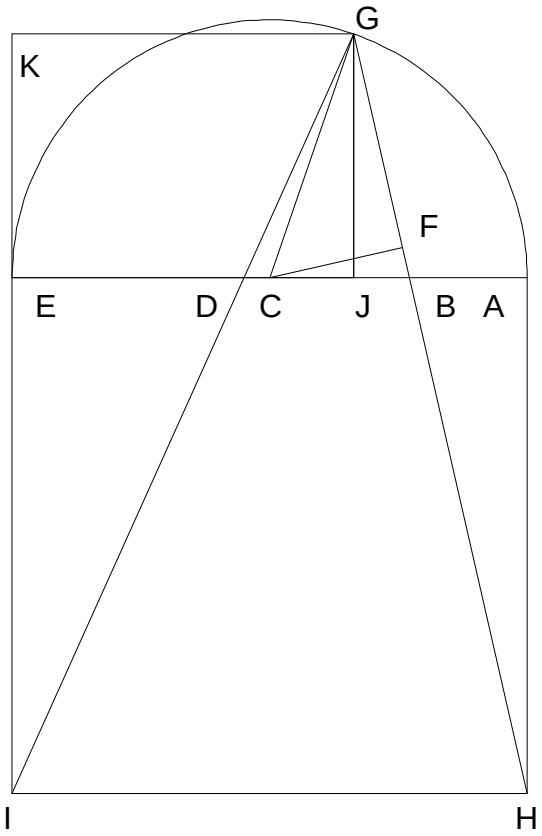
$GK_\delta := \frac{HI_\delta \cdot EG_\delta}{EH} \quad GP_\delta := \frac{AG_\delta \cdot GK_\delta}{AC + GK_\delta}$

$EP_\delta := \sqrt{(EG_\delta)^2 + (GP_\delta)^2}$

$CP_\delta := CE_\delta - EP_\delta \qquad CF_\delta := CD_\delta + DF$

$\underline{CP_\delta} - \sqrt{CD_\delta \cdot CF_\delta}$





10_31_94.MCD

Given AB & BE, divide BE such that BD + DE = BE and $BD = \sqrt{AB \cdot DE}$

The square in a right triangle on the hypotenuse is equal to the square of the remaining two segments (and all three squares taken to the point of similarity form a cubic relationship).

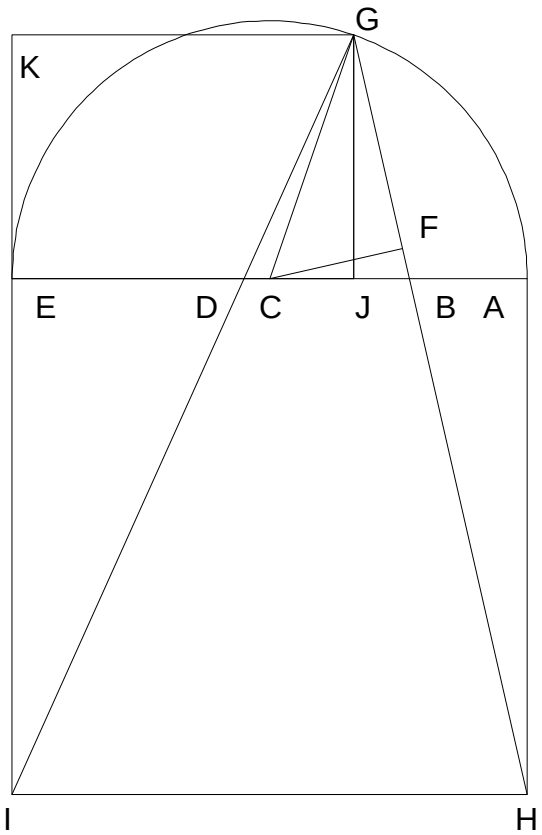
$$AE := AB + BE \quad AC := \frac{AE}{2}$$

$$AH := AE \quad EI := AE \quad BH := \sqrt{AB^2 + AH^2}$$

$$CG := AC \quad BC := AC - AB \quad BF := \frac{AB \cdot BC}{BH}$$

$$CF := \frac{AH \cdot BC}{BH} \quad FG := \sqrt{CG^2 - CF^2}$$

$$BG := BF + FG \quad BJ := \frac{AB \cdot BG}{BH}$$



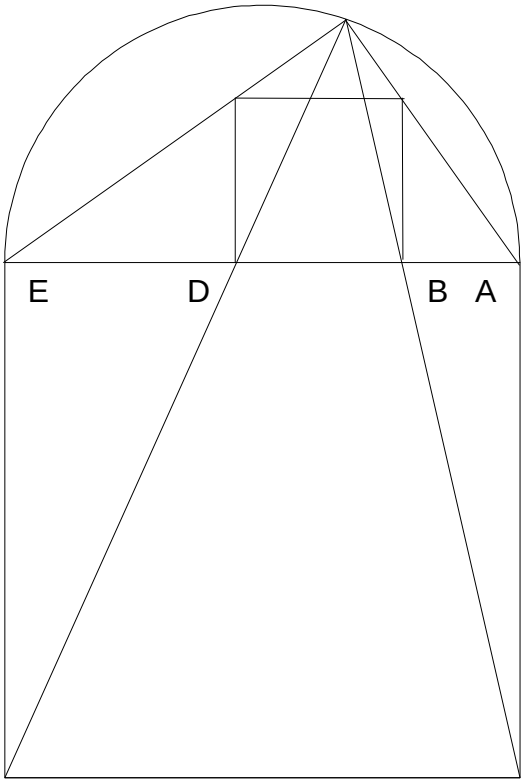
$$GJ := \frac{AH \cdot BJ}{AB} \quad AJ := AB + BJ$$

$$EJ := AE - AJ \quad GK := EJ \quad EK := GJ$$

$$IK := EI + EK \quad DE := \frac{GK \cdot EI}{IK}$$

$$BD := BE - DE$$

$$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB \cdot DE} = 0$$



Reducing the previous tautological chain to single equations for BD and DE.

$AB \equiv 20 \quad BE \equiv 12$

$$BD := \frac{\sqrt{AB}\cdot\left[-AB^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{AB + 4\cdot BE} + \sqrt{AB}\cdot BE \ldots + BE\cdot\sqrt{AB + 4\cdot BE}\right]}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

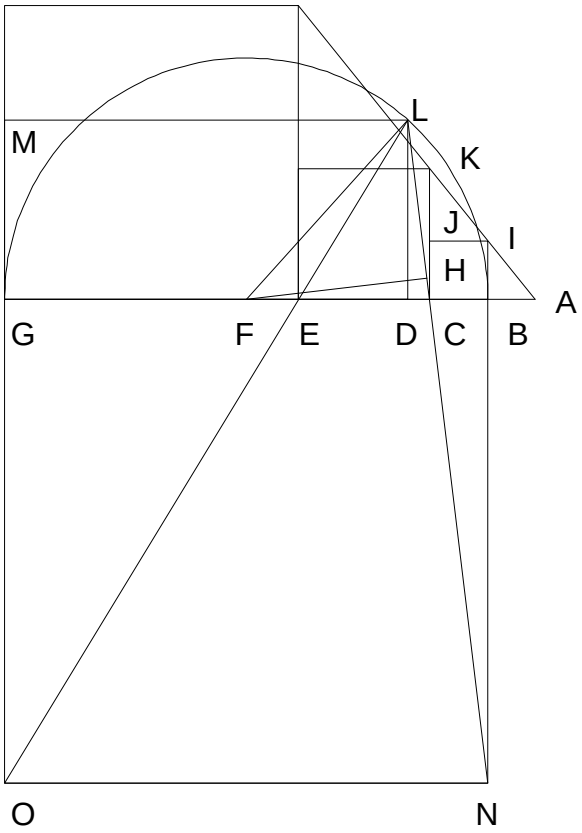
$BD = 8.439$

$$DE := \frac{AB^2 + 2\cdot AB\cdot BE - AB^{\left(\frac{3}{2}\right)}\cdot\sqrt{AB + 4\cdot BE} + 2\cdot BE^2}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

$DE = 3.561$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB\cdot DE} = 0$

Given AB & BE, BE has been divided such that BD + DE = BE and $BD = \sqrt{AB\cdot DE}$



11_01_94.MCD

Given AG - AB = BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BG > 3 BC.

$BG := 6 \quad BC := 1.9 \quad BN := BG$

$BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN}$

$FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN}$

$GM := DL \quad BD := BC + CD \quad DG := BG - BD$

$LM := DG \quad GO := BG \quad MO := GO + GM$

$EG := \frac{LM \cdot GO}{MO} \quad CG := BG - BC$

$CE := CG - EG \quad CJ := BC \quad CK := CE$

$IJ := BC \quad JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2}$

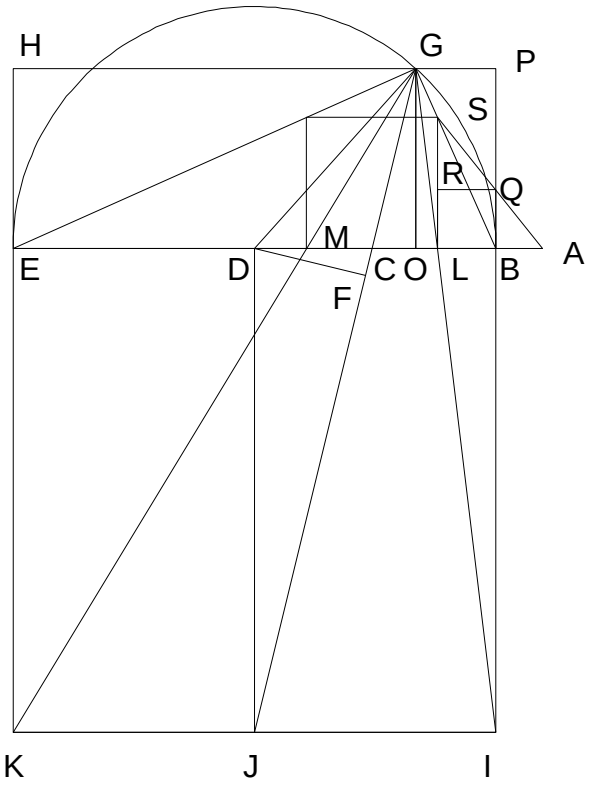
$BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE$

$AB = 36.723 \quad AG = 42.723 \quad AE = 40.621$

$\left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

Given AG - AB =BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, found was AB, AG, and $(AB \cdot AG^2)^{1/3}$.



Given AE - AB = BE and

$$\frac{(AB\cdot AE^2)^{\frac{1}{3}} - (AB^2\cdot AE)^{\frac{1}{3}}}{2} + (AB^2\cdot AE)^{\frac{1}{3}} - AE = BC,$$

find AB.

$BE := 70 \quad BC := 34 \quad BD := \frac{BE}{2} \quad BI := BE$

$DJ := BE \quad EK := BE \quad CD := BD - BC$

$CJ := \sqrt{DJ^2 + CD^2} \quad DF := \frac{DJ\cdot CD}{CJ}$

$DG := BD \quad FG := \sqrt{DG^2 - DF^2} \quad CF := \frac{CD\cdot DF}{DJ}$

$CG := FG - CF \quad CO := \frac{CD\cdot CG}{CJ}$

$DO := CD + CO$

$BO := BD - DO \quad GP := BO \quad GO := \frac{DJ\cdot CG}{CJ}$

$BP := GO \quad IP := BI + BP \quad BL := \frac{GP\cdot BI}{IP}$

$EH := GO \quad HK := EK + EH \quad EO := BE - BO$

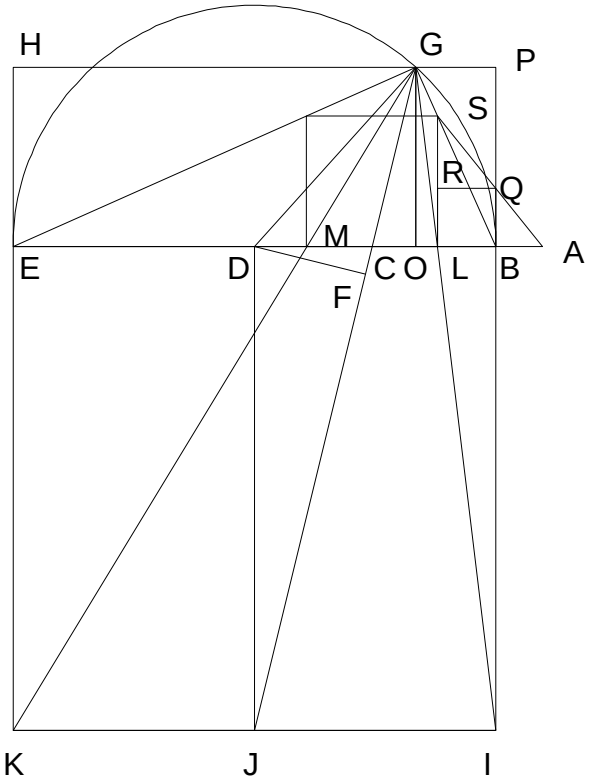
$GH := EO \quad EM := \frac{GH\cdot EK}{HK} \quad BM := BE - EM$

$BQ := BL \quad LR := BL \quad QR := BL$

$LM := BM - BL$

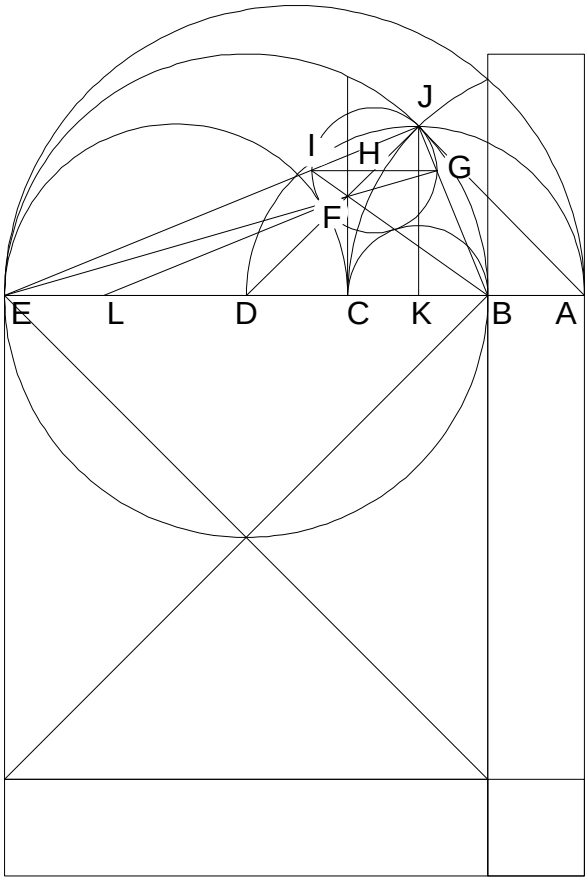
$LS := LM \quad RS := LS - LR \quad AB := \frac{QR\cdot BQ}{RS}$

$AE := AB + BE$



$AB = 510.028 \quad AE = 580.028 \quad BC = 34$

$$\frac{(AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}}{2} + (AB^2 \cdot AE)^{\frac{1}{3}} - AB = 34$$



Power Line At Square Root.

$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BE := 100 \quad AE_\delta := AB_\delta + BE$

$AC_\delta := \sqrt{AB_\delta \cdot AE_\delta} \quad BD := \frac{BE}{2} \quad DE := BD \quad DJ := BD$

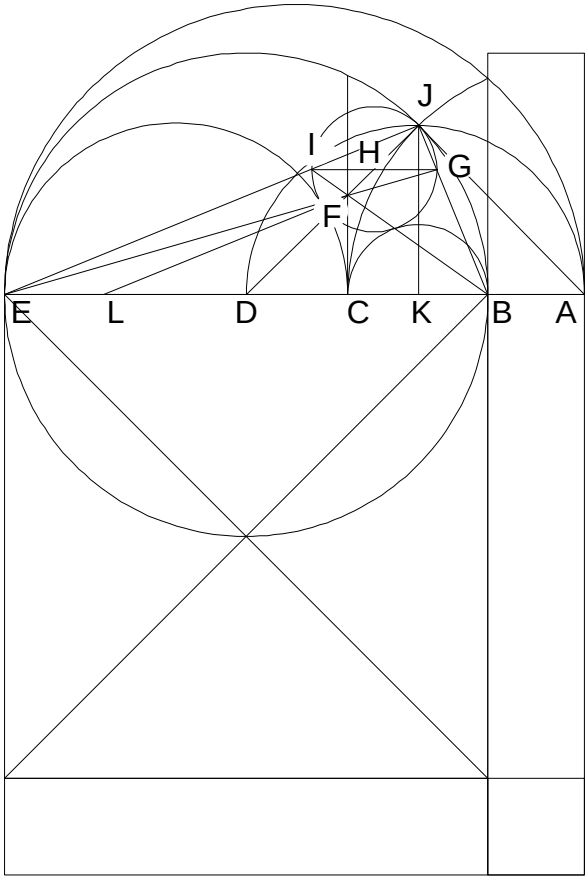
$AD_\delta := AB_\delta + BD \quad DK_\delta := \frac{DJ^2}{AD_\delta} \quad AK_\delta := AD_\delta - DK_\delta$

$BC_\delta := AC_\delta - AB_\delta \quad JK_\delta := \sqrt{AK_\delta \cdot DK_\delta} \quad CD_\delta := BD - BC_\delta$

$CF_\delta := \frac{JK_\delta \cdot CD_\delta}{DK_\delta} \quad BK_\delta := AK_\delta - AB_\delta \quad EK_\delta := BE - BK_\delta$

$CL_\delta := \frac{EK_\delta \cdot CF_\delta}{JK_\delta} \quad BL_\delta := BC_\delta + CL_\delta$

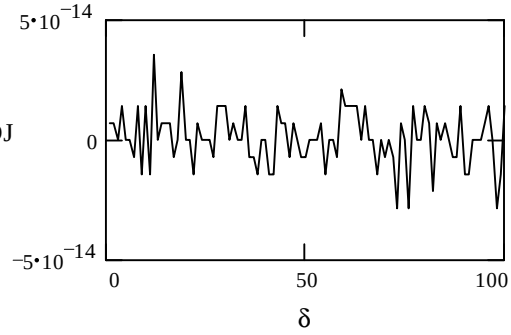
$BF_\delta := \sqrt{(BC_\delta)^2 + (CF_\delta)^2}$

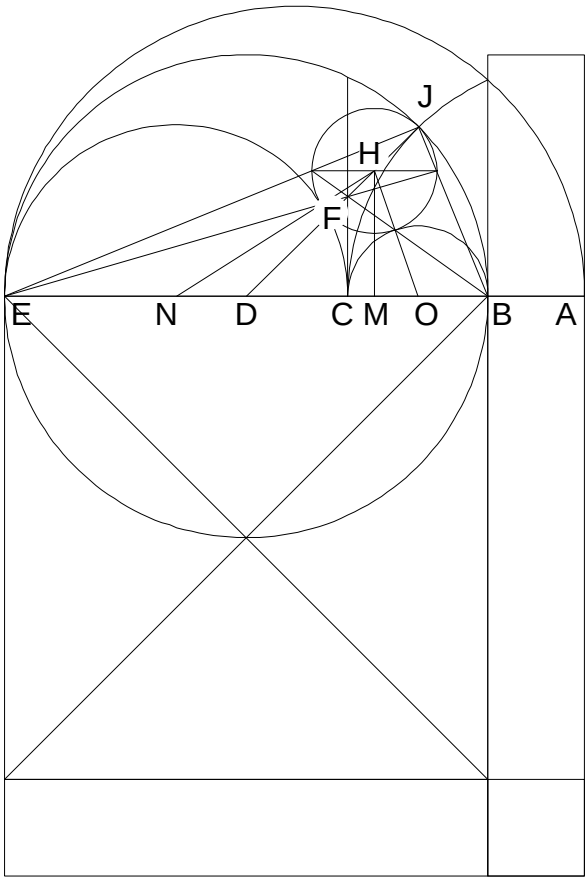


$BI_\delta := \frac{BF_\delta \cdot BE}{BL_\delta} \quad FI_\delta := BI_\delta - BF_\delta \quad GI_\delta := \frac{BE \cdot FI_\delta}{BF_\delta}$

$HJ_\delta := \frac{GI_\delta}{2} \quad DF_\delta := \sqrt{(CD_\delta)^2 + (CF_\delta)^2} \quad FH_\delta := \frac{DF_\delta \cdot GI_\delta}{BE}$

$\underline{(DF_\delta + FH_\delta + HJ_\delta)} - DJ$



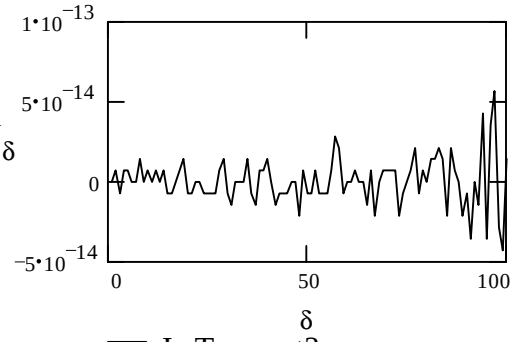


$DH_\delta := DF_\delta + FH_\delta \quad DM_\delta := \frac{CD_\delta \cdot DH_\delta}{DF_\delta} \quad CE_\delta := DE + CD_\delta$

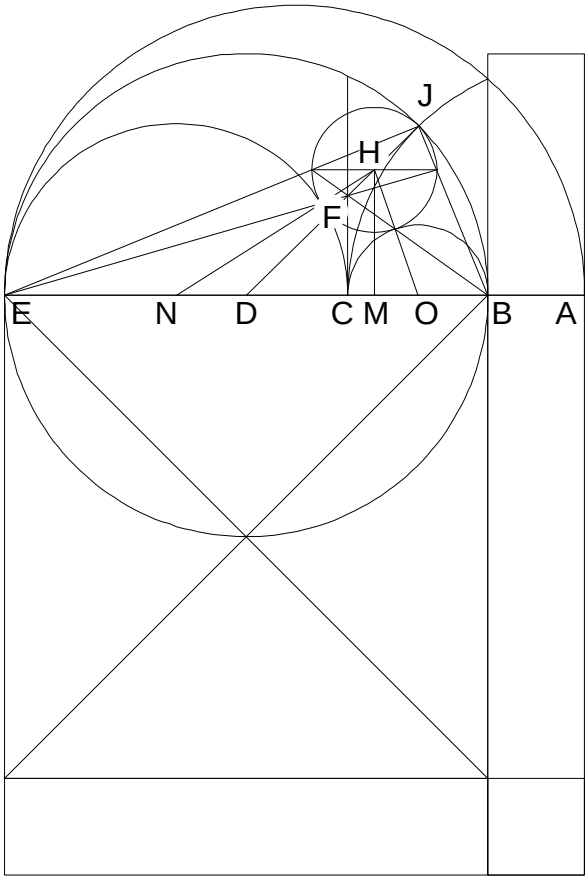
$CM_\delta := DM_\delta - CD_\delta \quad CN_\delta := \frac{CE_\delta}{2} \quad MN_\delta := CN_\delta + CM_\delta$

$HM_\delta := \frac{CF_\delta \cdot DH_\delta}{DF_\delta} \quad HN_\delta := \sqrt{(MN_\delta)^2 + (HM_\delta)^2}$

$\underline{(CN_\delta + HJ_\delta) - HN_\delta}$



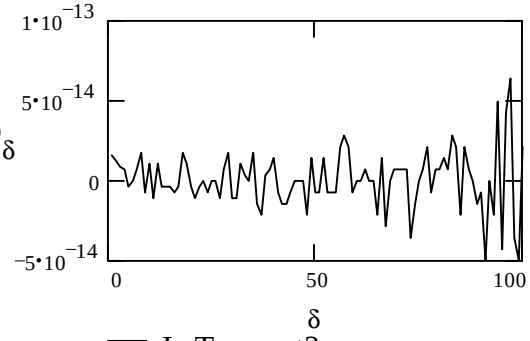
— Is Tangent?



$CO_\delta := \frac{BC_\delta}{2} \quad MO_\delta := CO_\delta - CM_\delta$

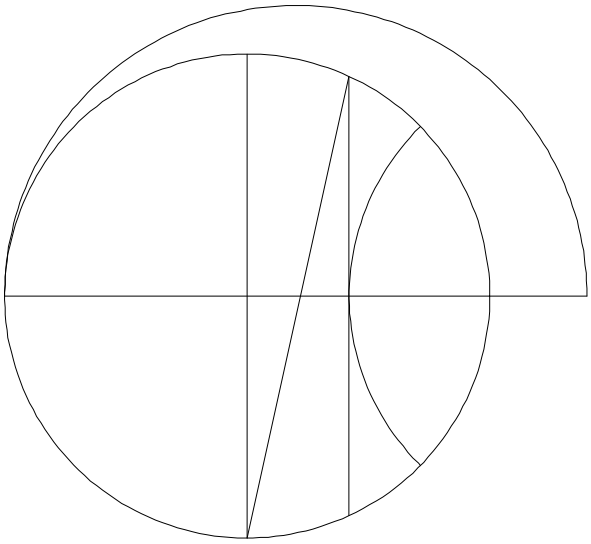
$HO_\delta := \sqrt{(HM_\delta)^2 + (MO_\delta)^2}$

$\underline{(CO_\delta + HJ_\delta) - HO_\delta}$

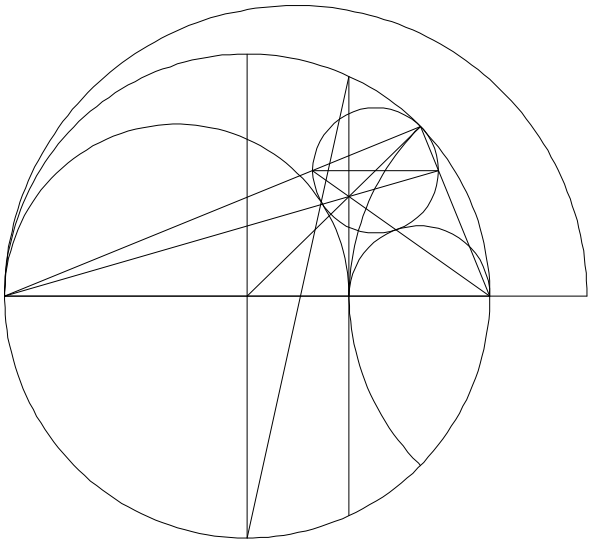


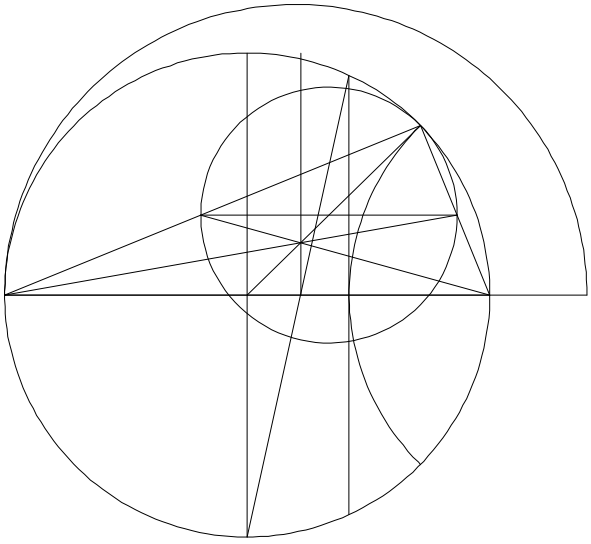
— Is Tangent?

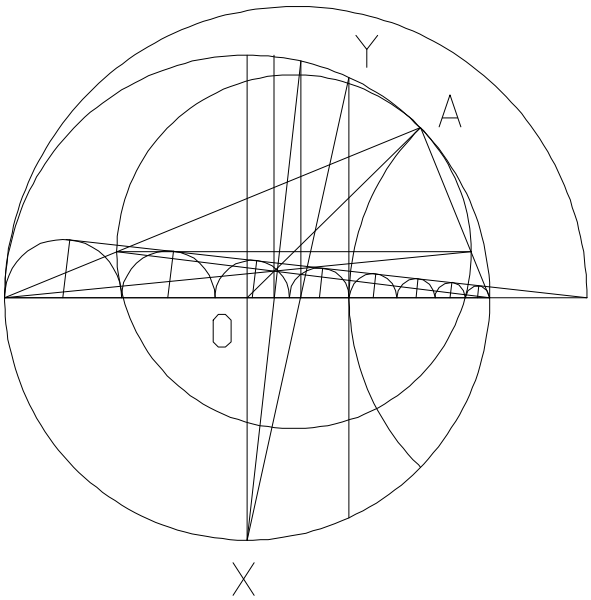
Two prime exponential series developed through power line progression.



I will present a series of plates to explain the process. The process can be infinitly repeated, supposing you had the tools to do it with.

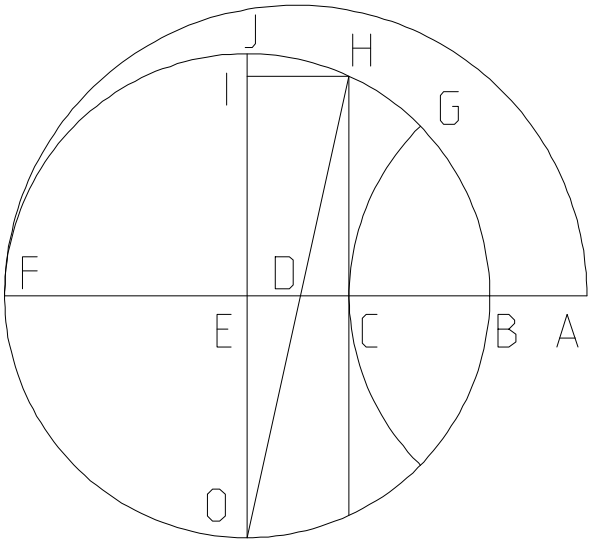






It is clear how OA uses the power line XY to provide a 2 prime exponential series.

Possible Problem: From a similarity point outside of a circle, place some 2 prine sequence of smaller circles on the larger circles diameter, all tangent in sequence.



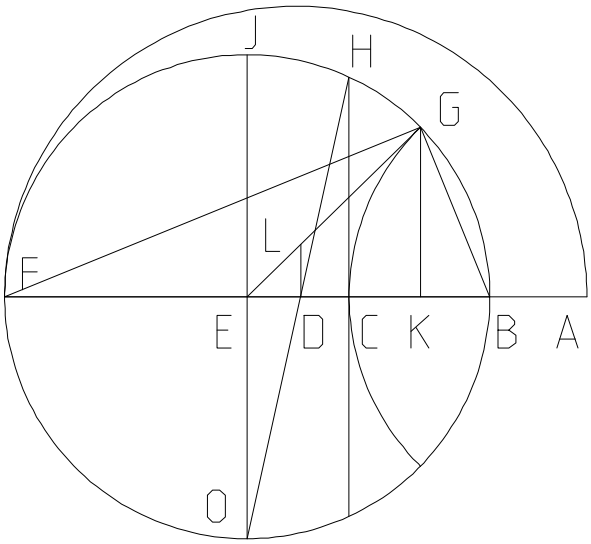
$AB := 1 \quad BF := 5 \quad AF := AB + BF$

$AC := \sqrt{AB \cdot AF} \qquad BE := \frac{BF}{2} \quad EO := BE$

$BC := AC - AB \qquad CF := BF - BC$

$CH := \sqrt{BC \cdot CF} \quad EI := CH \quad CE := BE - BC$

$HI := CE \quad IO := EO + EI \quad DE := \frac{HI \cdot EO}{IO}$

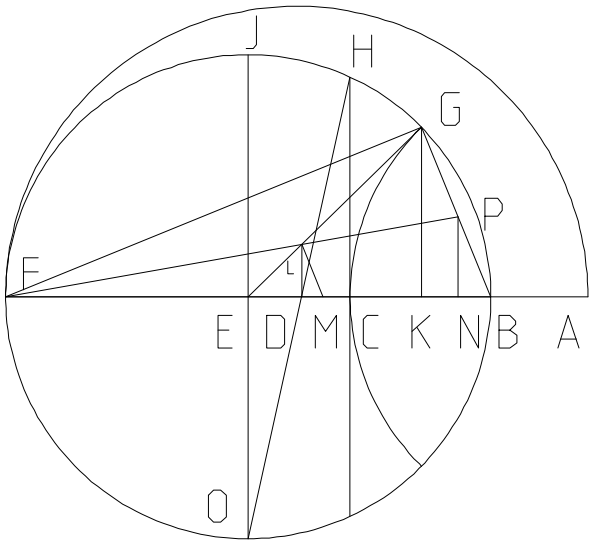


See 12_26_94.MCD for next equation.

$$GK := \frac{BF \cdot \left(\sqrt{AB} \cdot \sqrt{AB + BF}\right)}{(2 \cdot AB + BF)}$$

$EG := BE \quad EK := \sqrt{EG^2 - GK^2}$

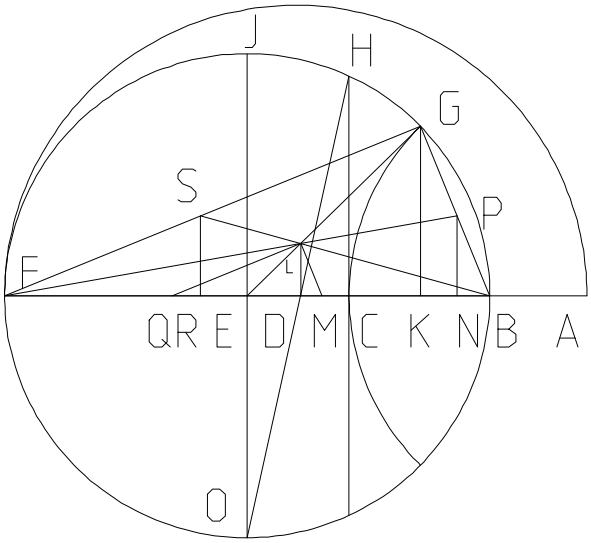
$DL := \frac{GK \cdot DE}{EK}$



$KN := BE - EK \quad DM := \frac{KN \cdot DE}{EK}$

$EF := BE \quad FM := EF + DM + DE$

$BN := \frac{DM \cdot BF}{FM} \quad NP := \frac{DL \cdot BF}{FM}$



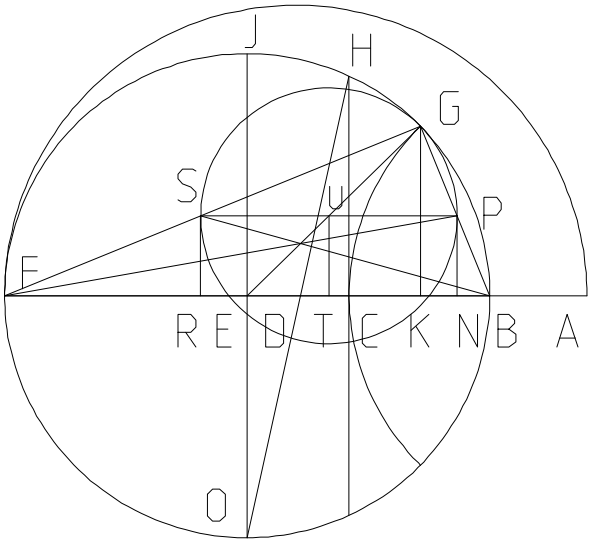
$KF := EK + EF \quad DQ := \frac{KF \cdot DL}{GK}$

$BD := BE - DE \quad BQ := BD + DQ$

$BR := \frac{BD \cdot BF}{BQ} \quad RS := \frac{DL \cdot BR}{BD}$

Are RS and NP equal?

$RS - NP = 0$



$TU := NP \quad ET := \frac{EK \cdot TU}{GK} \quad NR := BR - BN$

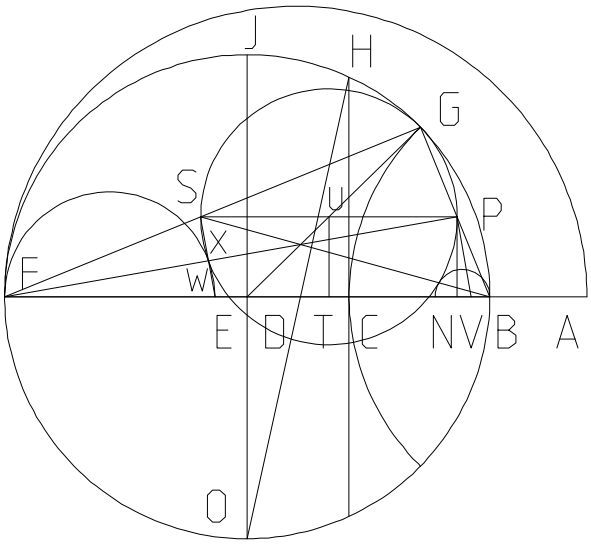
$EN := BE - BN \quad NT := EN - ET$

$PS := NR \quad PU := NT \quad EU := \sqrt{ET^2 + TU^2}$

Is NT half of NR? $\frac{NR}{NT} = 2$

Does GU = PU? $GU := EG - EU$

$GU - PU = 0$



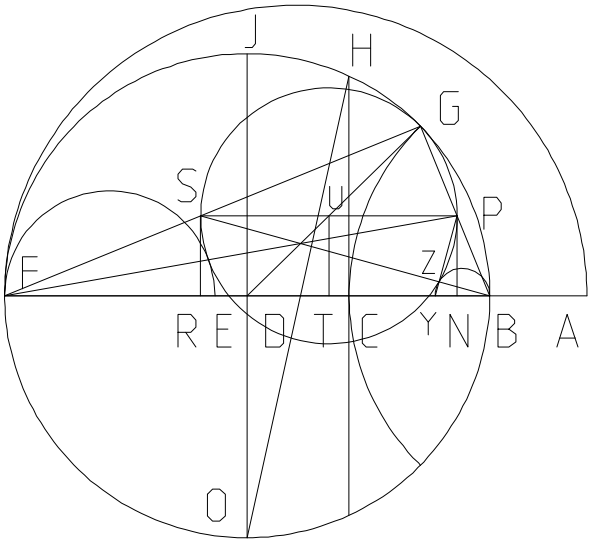
$BT := BN + NT \quad FN := BF - BN \quad FP := \sqrt{NP^2 + FN^2}$

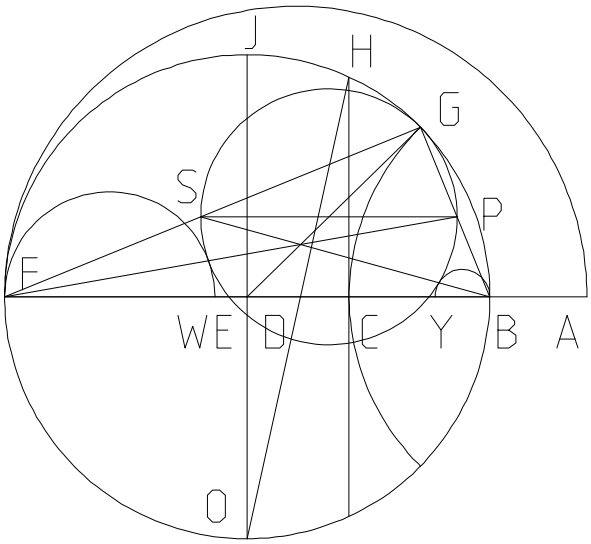
$FV := \frac{FP^2}{FN} \quad PX := \frac{FP \cdot PS}{FV} \quad FX := FP - PX$

$FW := \frac{FV \cdot FX}{FP} \quad BW := BF - FW \quad AW := AB + BW$

Is AW a quarter root of AB, AF?

$\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AW = 0$





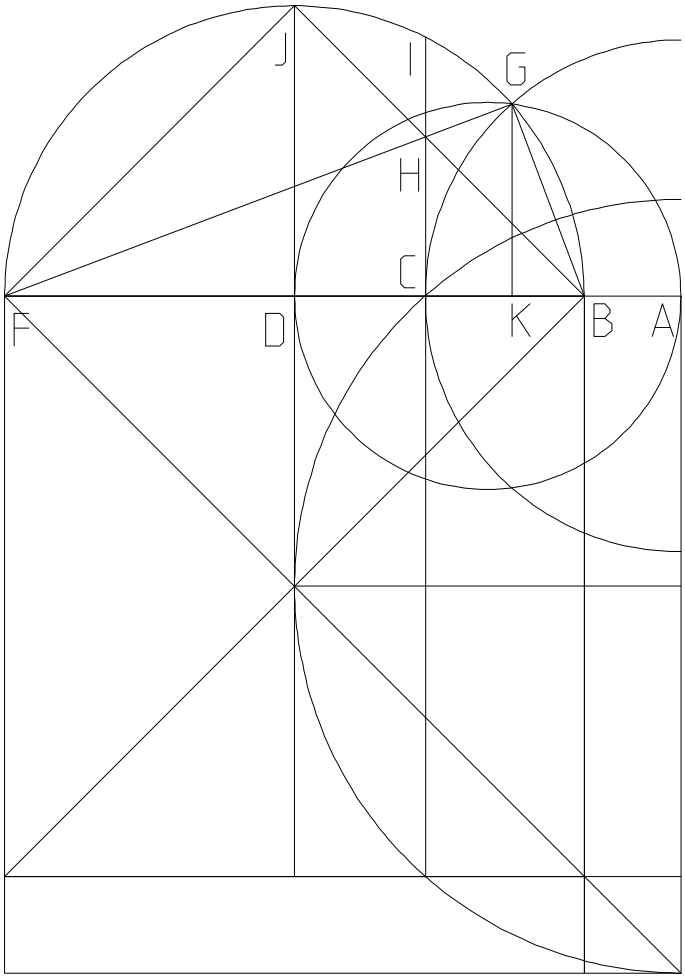
$BS := \sqrt{BR^2 + RS^2}$ $SZ := \frac{BR \cdot PS}{BS}$

$BZ := BS - SZ$ $BY := \frac{PS \cdot BZ}{SZ}$

$AY := AB + BY$

Is AY a quarter root of AB, AF?

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AY = 0$



Does H and G have a constant relationship?

$\delta := 1 .. 1000 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$BF := 6 \quad BD := \frac{BF}{2} \quad AF_\delta := AB_\delta + BF$

$AC_\delta := \sqrt{AB_\delta \cdot AF_\delta} \quad DJ := BD \quad DF := BD$

$BC_\delta := AC_\delta - AB_\delta \quad CD_\delta := BD - BC_\delta$

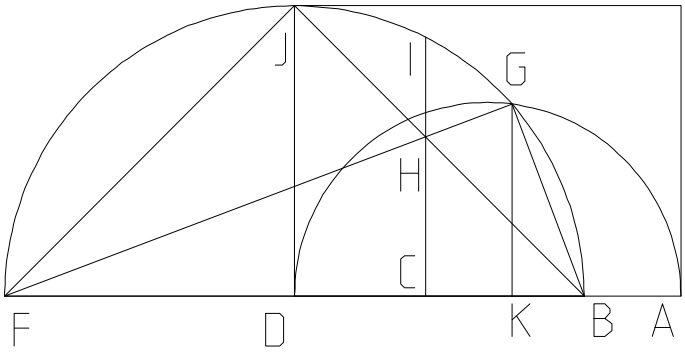
$CH_\delta := \frac{DJ \cdot BC_\delta}{BD} \quad AG_\delta := AC_\delta$

$AD_\delta := AB_\delta + BD \quad AK_\delta := \frac{(AG_\delta)^2}{AD_\delta}$

$GK_\delta := \sqrt{(AG_\delta)^2 - (AK_\delta)^2}$

$BK_\delta := AK_\delta - AB_\delta \quad CK_\delta := BC_\delta - BK_\delta$

$KF_\delta := CK_\delta + CD_\delta + DF$

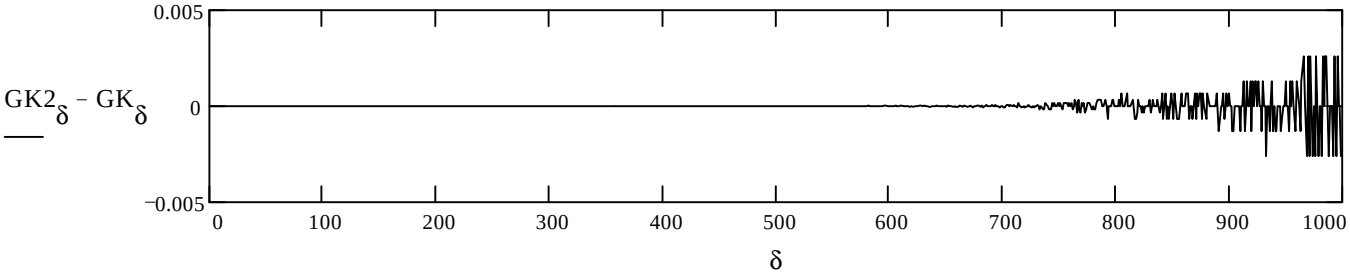


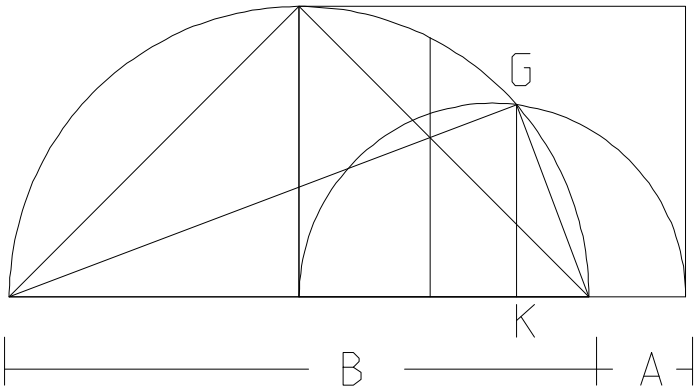
Does FH and FG have identical slopes?

$CF_\delta := CD_\delta + DF$

$GK2_\delta := \frac{CH_\delta \cdot KF_\delta}{CF_\delta}$

Therefore G and H are constantly co-linear.





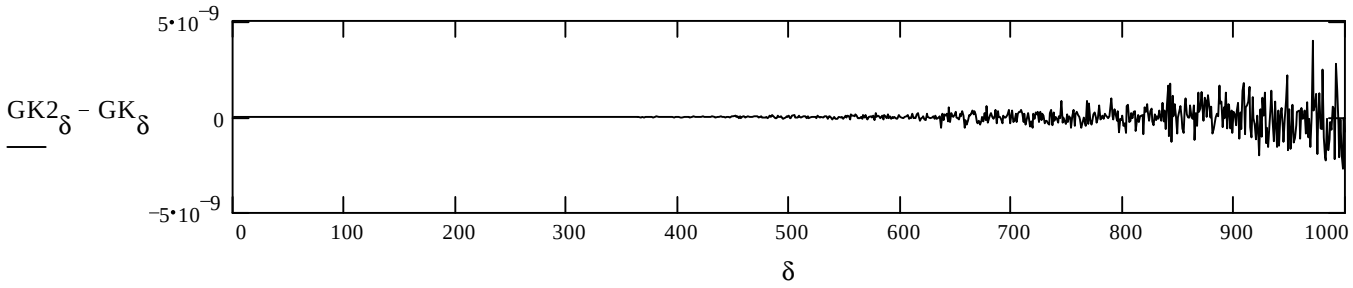
Thus this file can be redone as: "Given BC = $\sqrt{AB \cdot AF}$ and BF, find AB."

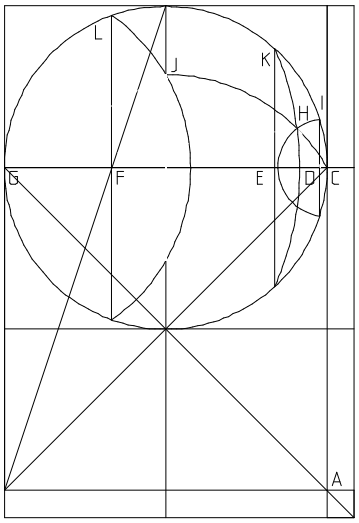
The Formula for GK vs. GK2 demonstrates that the symbolic processor cannot always resolve to simplest form. GK2 is the processors final attempt. An attempt with Mathcad 6 gives the same result.

$A_\delta := AB_\delta \quad B := BF$

$$GK2_\delta := B \cdot \frac{\left[\left(A_\delta\right)^{\left(\frac{3}{2}\right)} \cdot \sqrt{A_\delta + B} - \left(A_\delta\right)^2 + B \cdot \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} - B \cdot A_\delta \right]}{\left[\left(2 \cdot A_\delta + B\right) \cdot \left(B - \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} + A_\delta\right) \right]}$$

$$GK_\delta := \frac{B \cdot \left(\sqrt{A_\delta} \cdot \sqrt{A_\delta + B}\right)}{\left(2 \cdot A_\delta + B\right)}$$





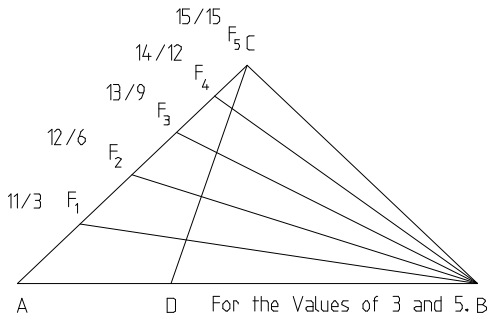
One Circle

By John Clark

One Square

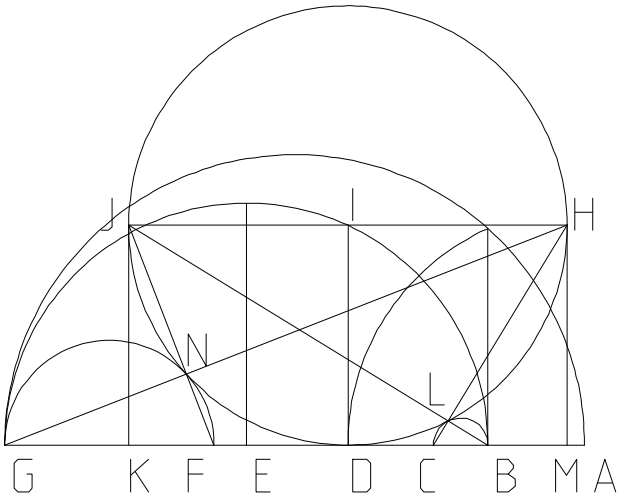
One Line

And the Delian Quest



1995

Alternate method for Quad Roots



$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BG := 10 \quad AG_\delta := AB_\delta + BG$

$AD_\delta := \sqrt{AB_\delta \cdot AG_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

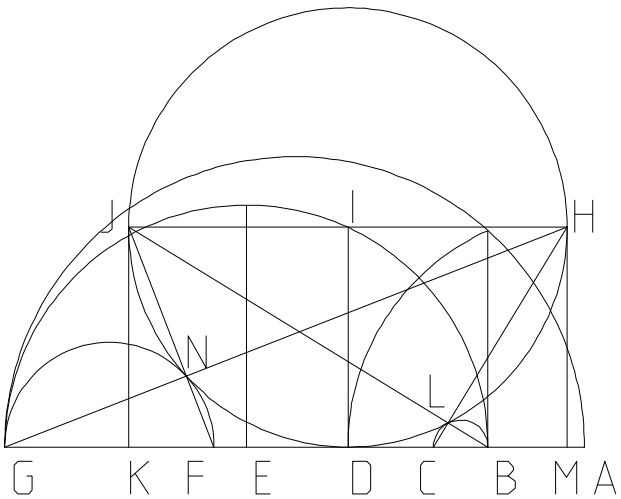
$DG_\delta := BG - BD_\delta \quad DI_\delta := \sqrt{BD_\delta \cdot DG_\delta}$

$HI_\delta := DI_\delta \quad IJ_\delta := DI_\delta \quad HJ_\delta := HI_\delta + IJ_\delta$

$DK_\delta := DI_\delta \quad JK_\delta := DI_\delta \quad BK_\delta := BD_\delta + DK_\delta$

$BJ_\delta := \sqrt{\left(BK_\delta\right)^2 + \left(JK_\delta\right)^2} \quad JL_\delta := \frac{BK_\delta \cdot HJ_\delta}{BJ_\delta}$

$BL_\delta := BJ_\delta - JL_\delta \quad BC_\delta := \frac{HJ_\delta \cdot BL_\delta}{JL_\delta}$



$HM_\delta := DI_\delta \quad DM_\delta := DI_\delta \quad MG_\delta := DM_\delta + DG_\delta$

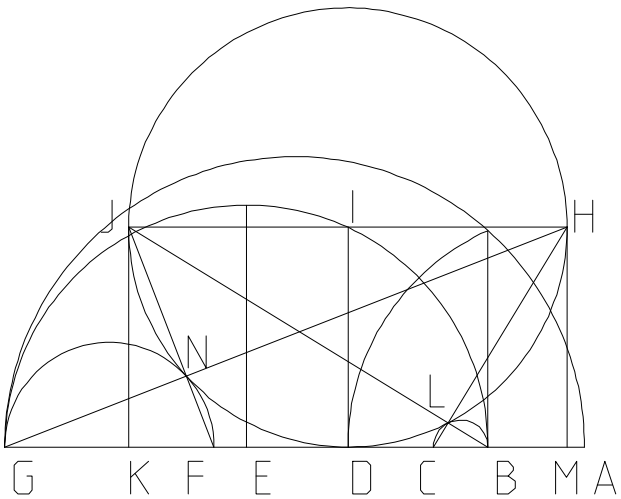
$GH_\delta := \sqrt{\left(MG_\delta\right)^2 + \left(HM_\delta\right)^2} \quad HN_\delta := \frac{MG_\delta \cdot HJ_\delta}{GH_\delta}$

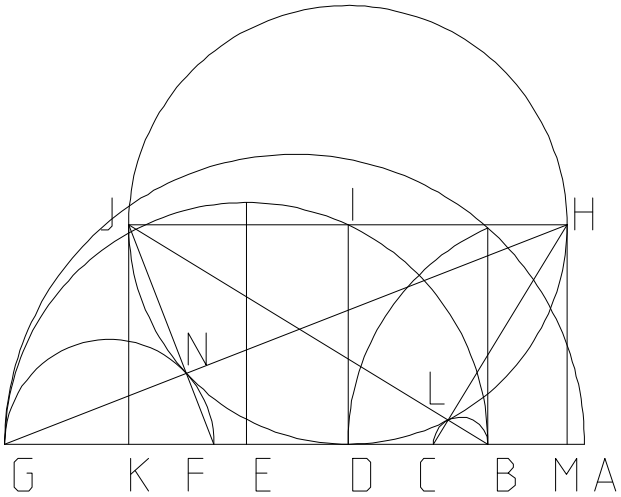
$GN_\delta := GH_\delta - HN_\delta \quad FG_\delta := \frac{HJ_\delta \cdot GN_\delta}{HN_\delta}$

$BF_\delta := BG - FG_\delta$

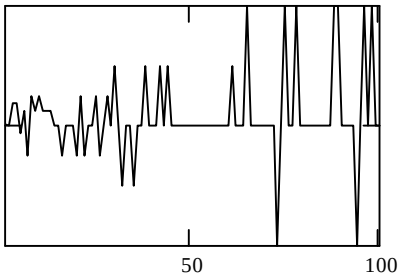
$AF_\delta := AB_\delta + BF_\delta$

$AC_\delta := AB_\delta + BC_\delta$



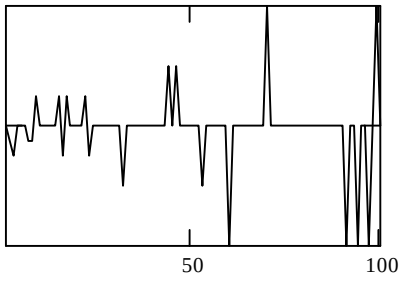


$$\left[AC_{\delta} - \left[\left(AB_{\delta} \right)^{3} \cdot AG_{\delta} \right]^{\frac{1}{4}} \right]_{0}$$



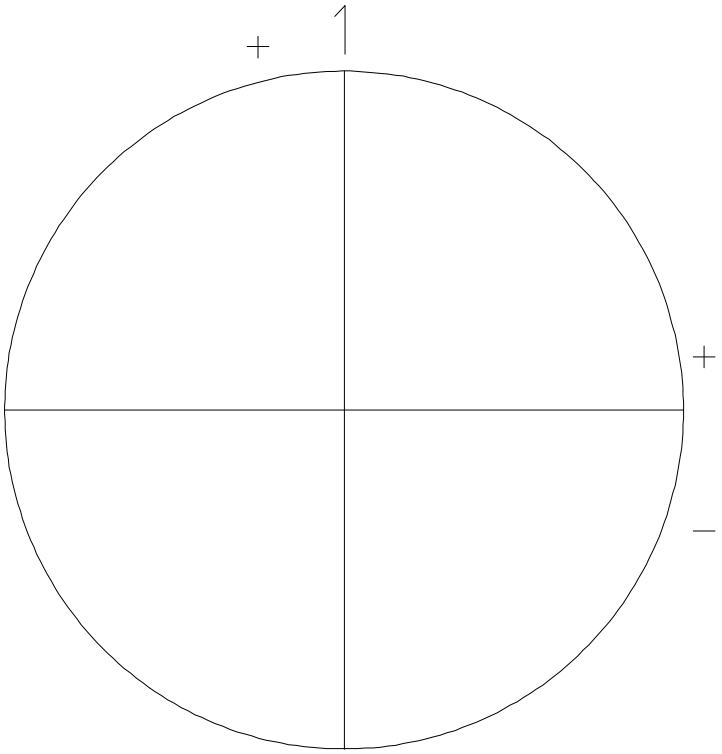
δ

$$\left[AF_{\delta} - \left[AB_{\delta} \cdot \left(AG_{\delta} \right)^{3} \right]^{\frac{1}{4}} \right]_{0}$$



δ

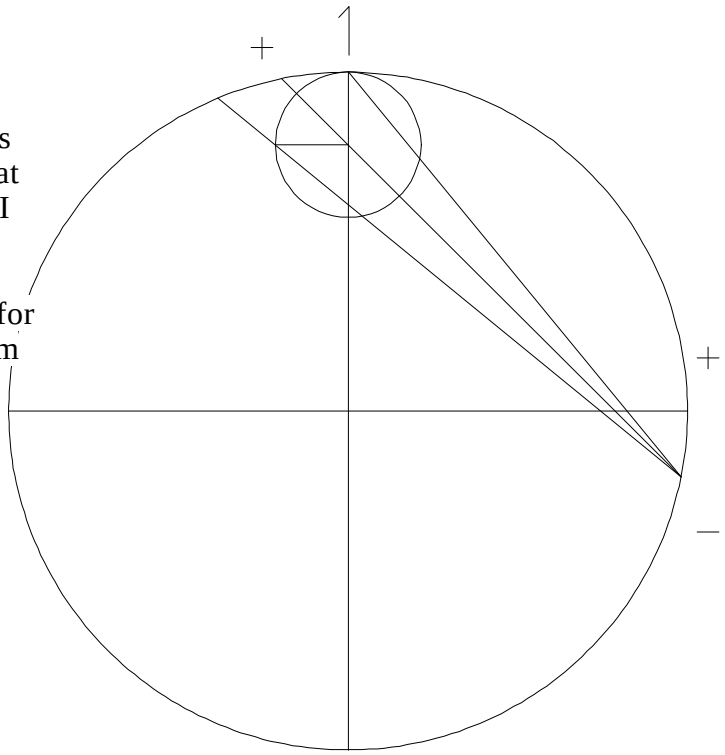
The symbolic processor on my computer could not reduce the chain to the final equations.



Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

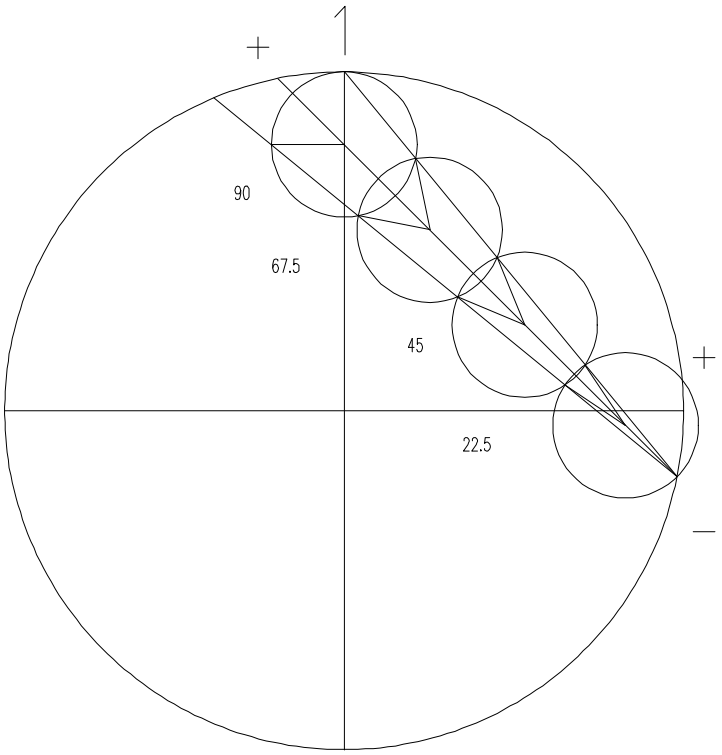
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$



$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

$8 + 1 - 1 = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 - 2 = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 - 2 - 2 = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 - 2 - 2 - 2 = 2$	$2 \cdot 11.25 = 22.5$
$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$	

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

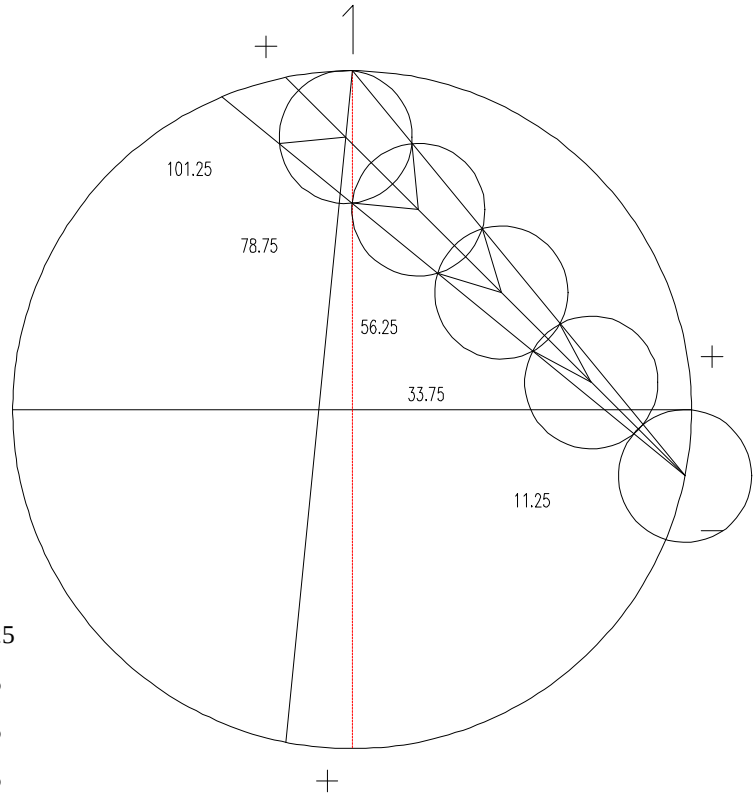
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

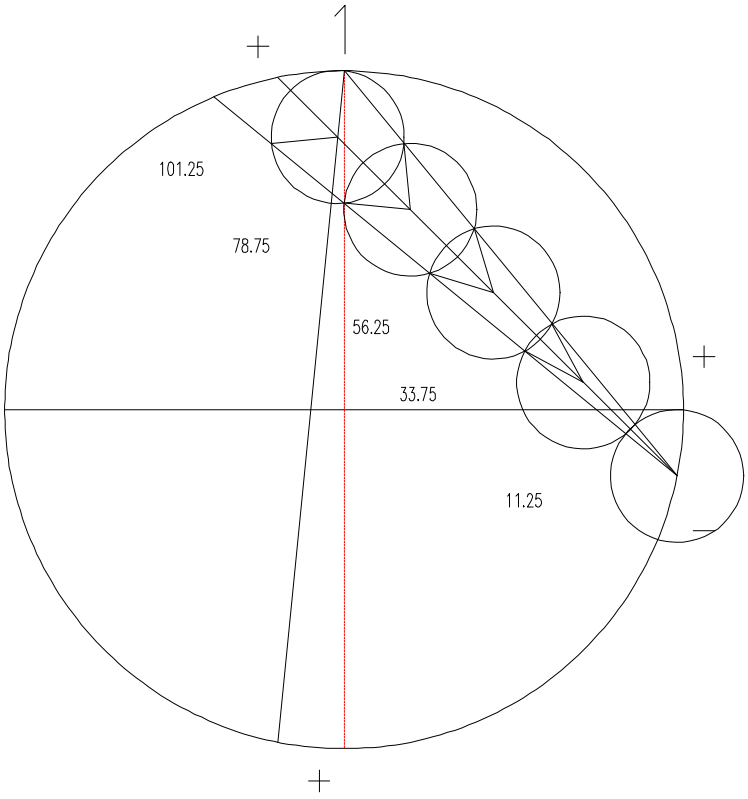
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$



$8 + 1 + 1 - 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 + 1 - 1 - 2 = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 + 1 - 1 - 2 - 2 = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

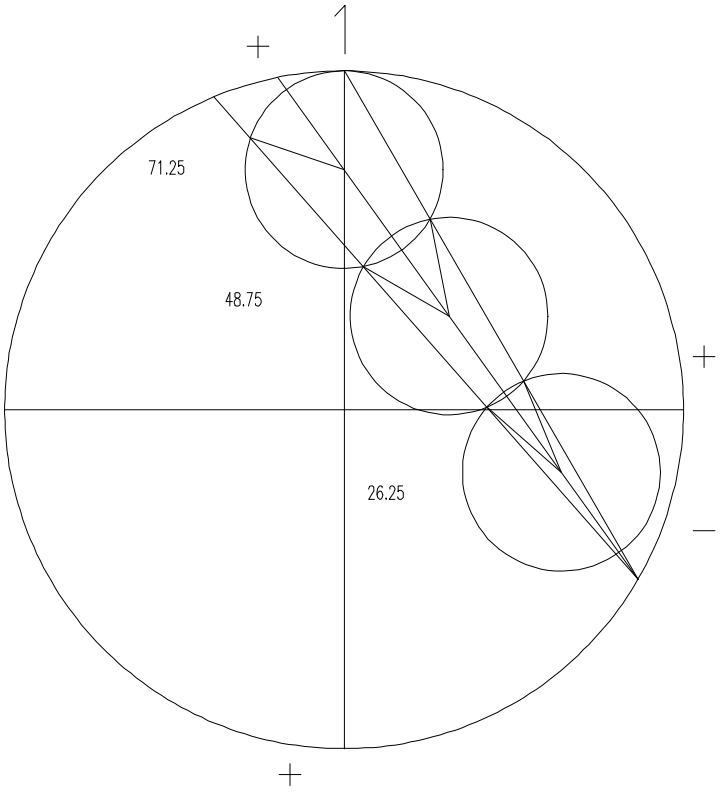
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 - (1 \cdot 2) = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 - (2 \cdot 2) = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 - (3 \cdot 2) = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 - (4 \cdot 2) = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1, 2) = 1$



$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

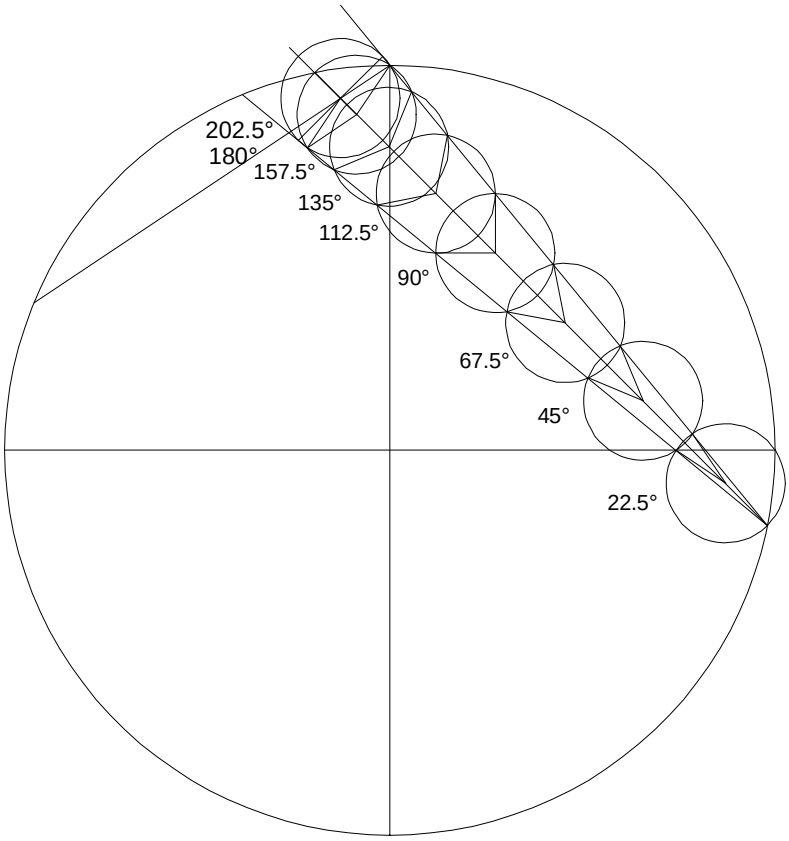
$(24 + 3) - 8 = 19$	$19 \cdot 3.75 = 71.25$
$(24 + 3) - 8 - (1 \cdot 6) = 13$	$13 \cdot 3.75 = 48.75$
$(24 + 3) - 8 - (2 \cdot 6) = 7$	$7 \cdot 3.75 = 26.25$
$(24 + 3) - 8 - (3 \cdot 6) = 1$	$1 \cdot 3.75 = 3.75$

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$



$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

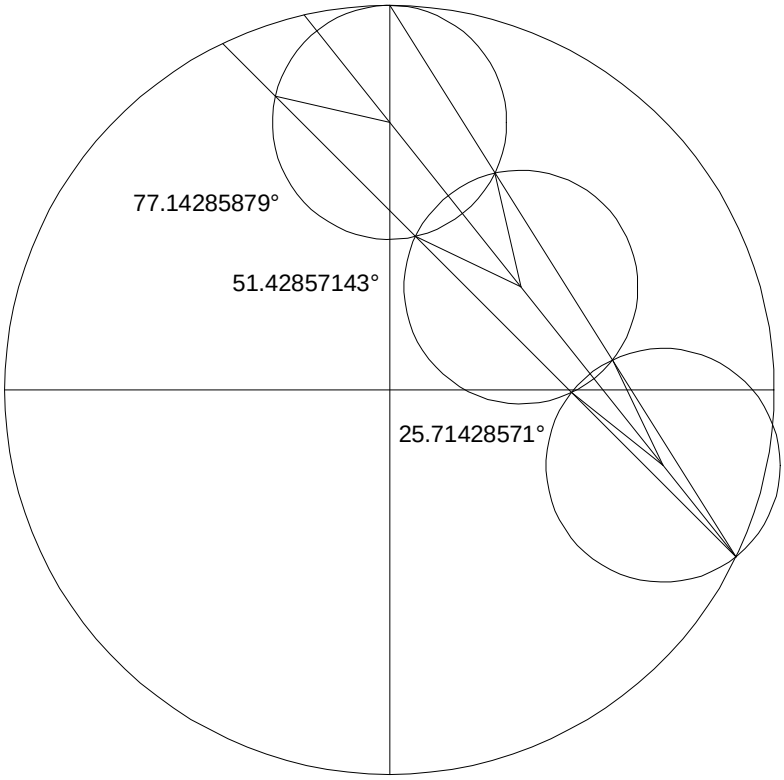
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

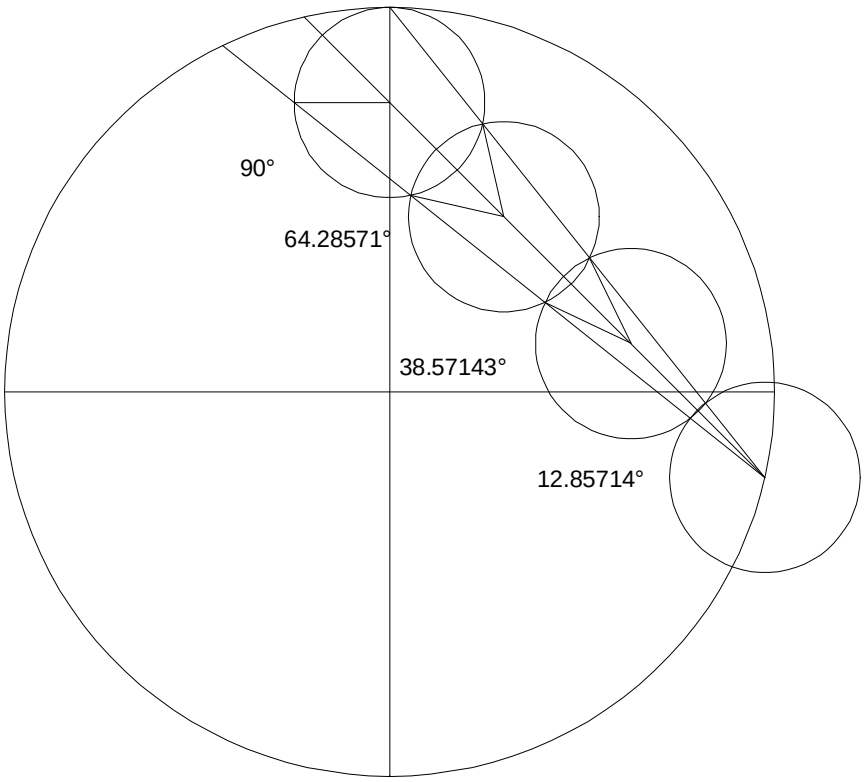
$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$



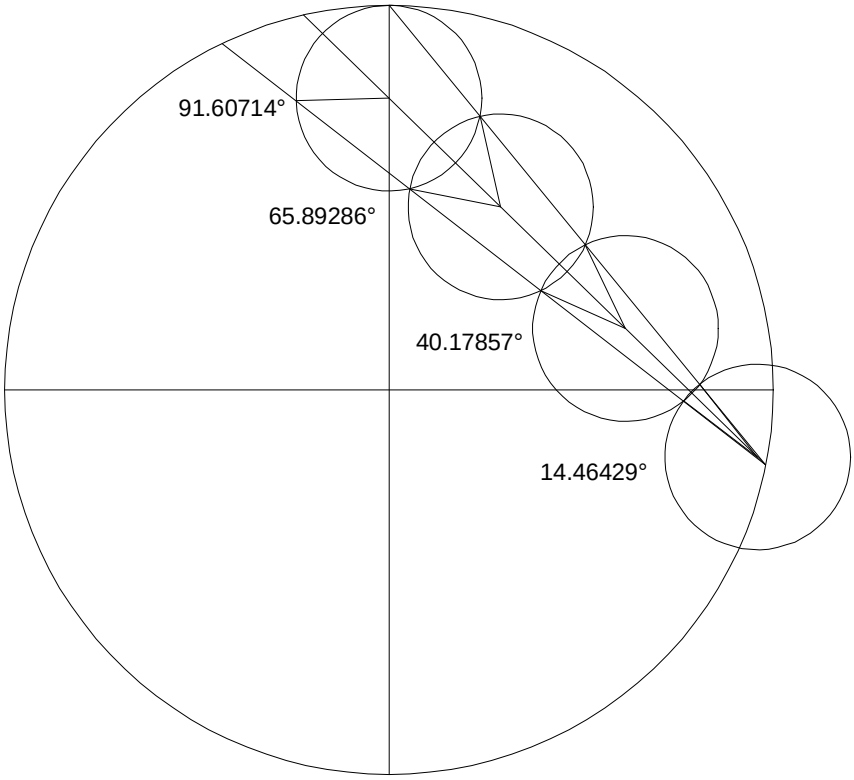
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$



$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

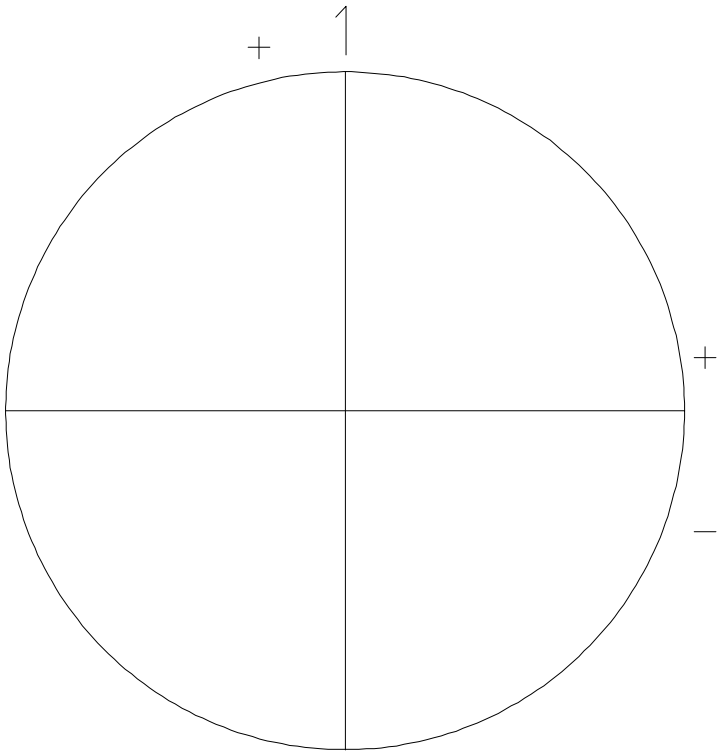
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

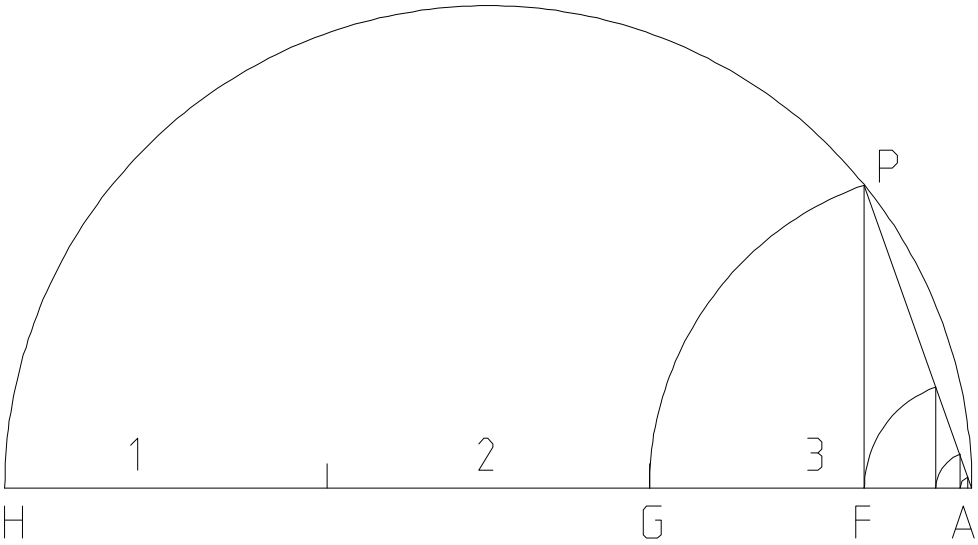
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$



Work in progress.



Exponential Progressions. 04_01_95

If I want to multiply any number by any power, this is the a geometric process for doing so.

The given figure is drawn for the third power of 3.

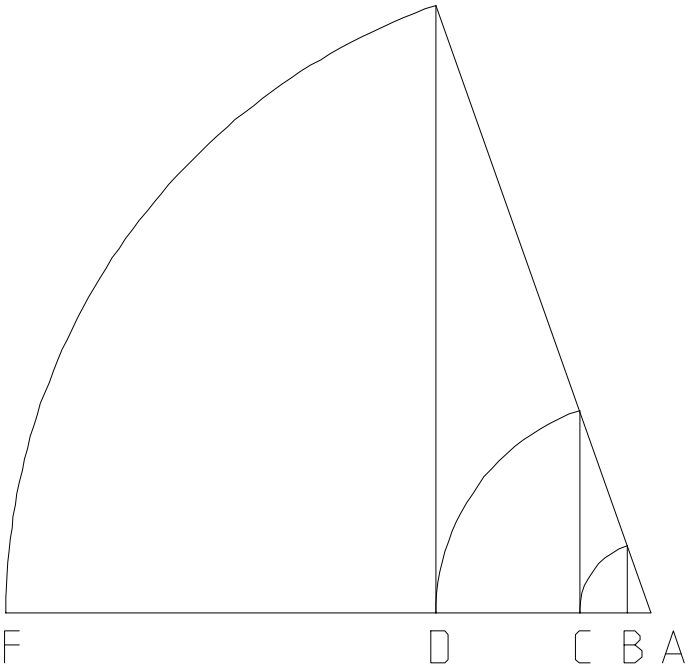
The third division between A and F is very hard to see. BS = Base Segments

$AH := 10 \qquad \delta := 0..10 \qquad BS := 8$

$AP := \frac{AH}{BS} \qquad AP = 1.25$

$AG_0 := \frac{(AP)^2}{AH} \qquad AF_0 := AG_0 \qquad \Delta := 2..10$

$\begin{pmatrix} AF_{\delta+1} \\ AG_{\delta+1} \end{pmatrix} := \begin{bmatrix} AG_\delta \\ \frac{AP \cdot AG_\delta}{AH} \end{bmatrix} \qquad AB_\delta := \frac{AG_1}{AF_\delta}$

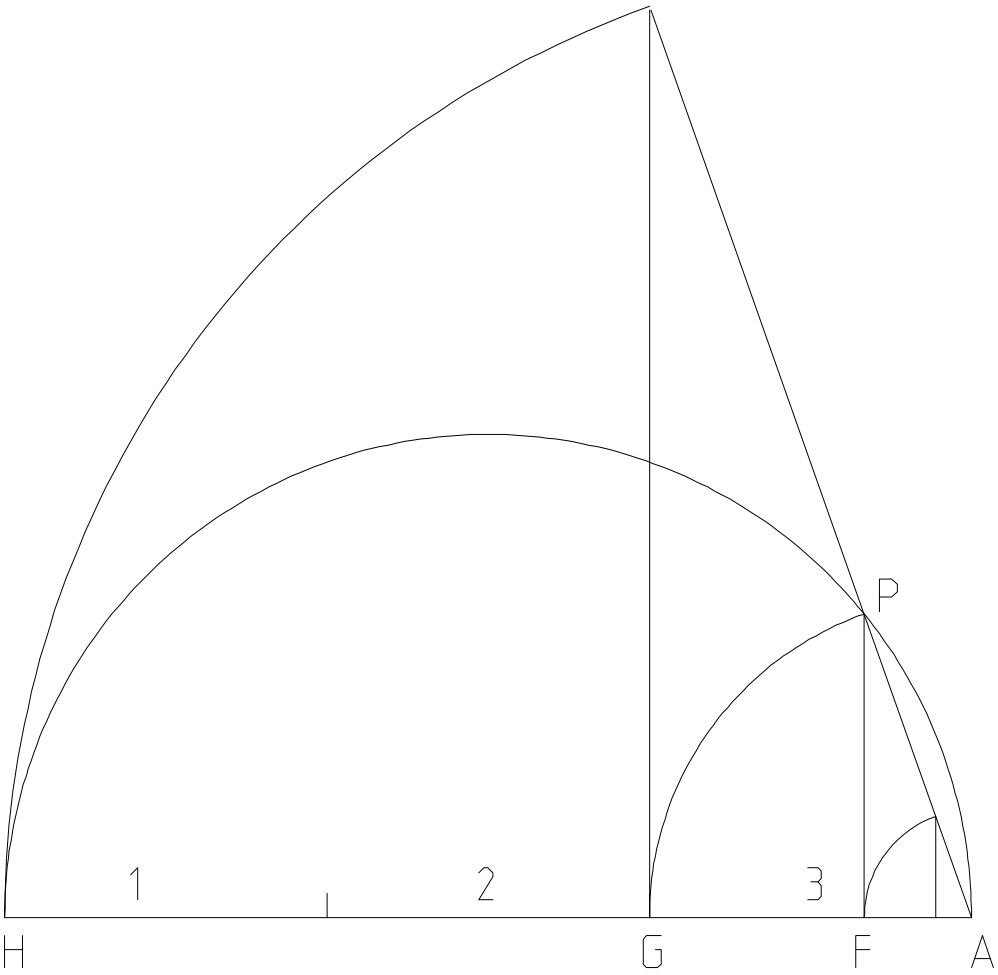


δ	AB_Δ	BS^δ
0	1	1
1	8	8
2	64	64
3	512	512
4	4096	4096
5	32768	32768
6	262144	262144
7	2097152	2097152
8	16777216	16777216
9		134217728
10		$1.07374182 \cdot 10^9$

Making the number of divisions 3, provides 3 cube result. AB divides AF 27 times. Etc. It can be seen that using a normal straight edge and compass one needs a very large piece of paper to work this.

You will notice that I took only one of the possible two divisions from which to project from. The other would be 2/3. At 2/3 my unit divisions would still be 27, but now AB would take up 2 cube of them, or AB would be 8 units.

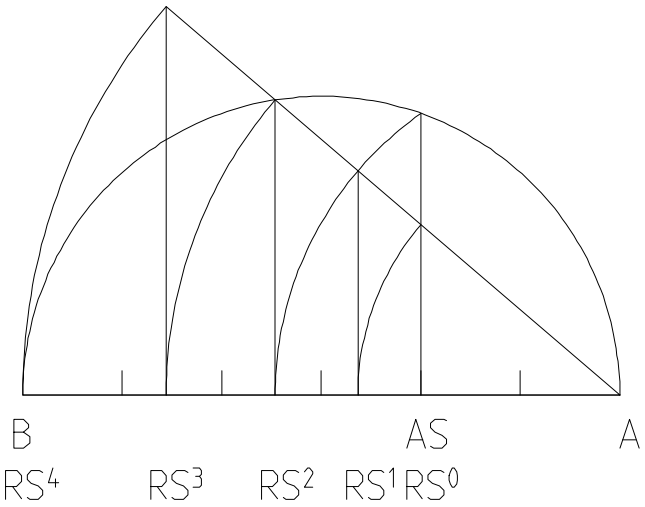
For an 8 cube series then, the value for AB would be 1 of 512, 8 of 512, 27 of 512, 64 of 512, 125 of 512, 216 of 512, 343 of 512



Since the slope is established, one can take the first two divisions and utilizes the whole figure.

$E := 1 .. BS \qquad ROOT := 3$

E	$\frac{BS^{ROOT}}{E^{ROOT}}$	E^{ROOT}
1	512	1
2	64	8
3	18.96296296	27
4	8	64
5	4.096	125
6	2.37037037	216
7	1.49271137	343
8	1	512



About The Laws of Exponents and Ratios

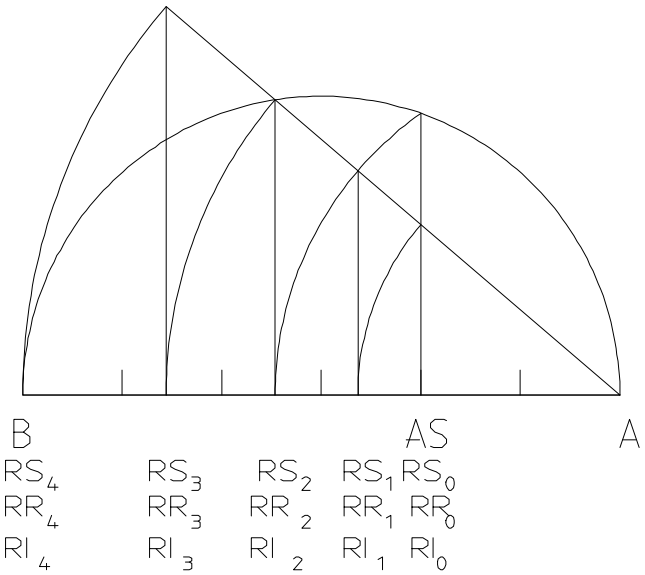
04_22_95.MCD

$\Delta := 22 \quad \delta := 1 .. \Delta \quad AB := 7$

Base Segments = BS $BS := 99$

Base Index = BI $BI := 13$

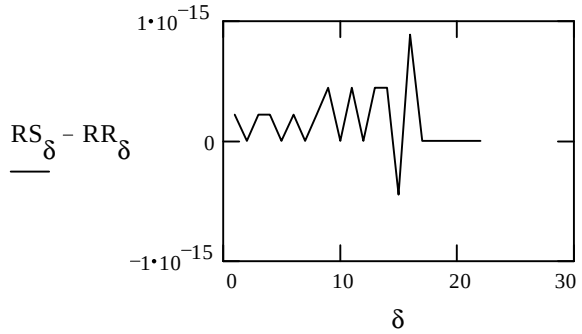
Root Series = RS $RS_\delta := \left[\left(\frac{BI}{BS} \cdot AB \right)^{\Delta - \delta} \cdot AB^{\delta} \right]^{\frac{1}{\Delta}}$



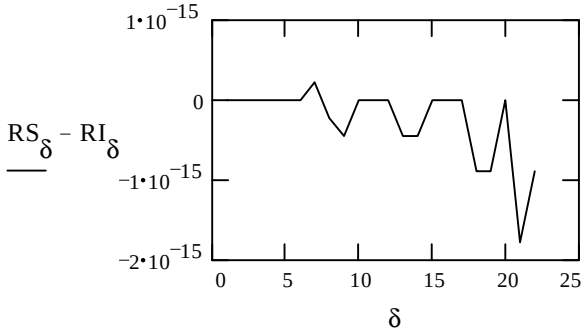
Root Series By Ratio = RR $RR_\delta := \left(\frac{BI}{BS} \right)^{\frac{\Delta - \delta}{\Delta}} \cdot AB$

Root Series By Inverse Ratio = RI

$RI_\delta := \left(\frac{BS}{BI} \right)^{\frac{\delta}{\Delta}} \cdot \left(\frac{BI}{BS} \cdot AB \right)$



$\sum_\delta \left(RS_\delta - RR_\delta \right) = 0$

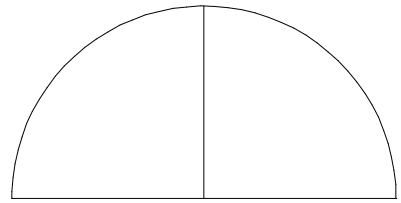


$\sum_\delta \left(RS_\delta - RI_\delta \right) = 0$

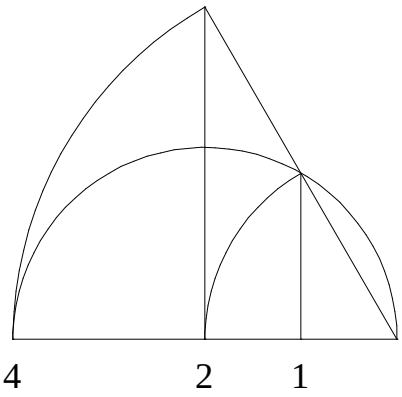
On the concept of unit and universe of discourse:

Euclidean exponentiation provides a good example with which to demonstrate the distinction between the concepts of unit and the universe of discourse.

Taking 2 for the *universe* of discourse would be represented graphically as:

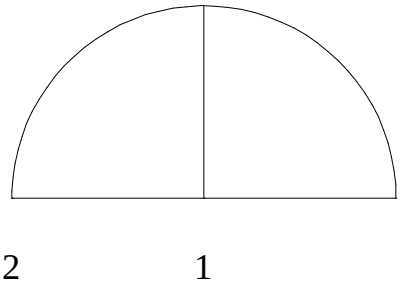


To represent 2^2 within this universe of discourse one would draw:

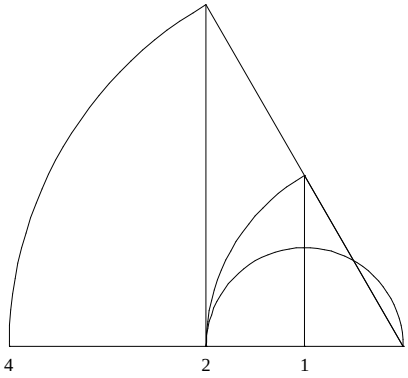


Our original 2 is divided into a 4^{th} segment.

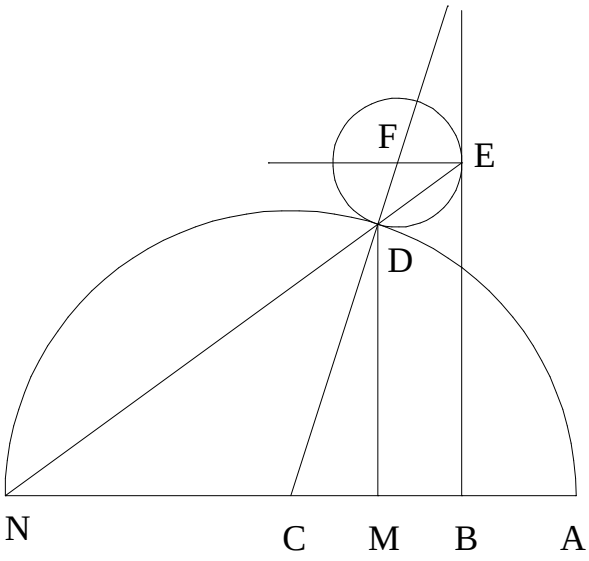
Now if 2 is taken as the *unit* of discourse we may still represent it as;



however to represent 2^3 now would be drawn as



One will notice that in example 1, the unit changed while the universe remained, in example 2 the unit remained while the universe changed. One could call example 1 an example of deduction and example 2 an example of induction.



09_13_95.MCD

A Study in Placement.

Given AC, AB, AM find EF.

Segment A.

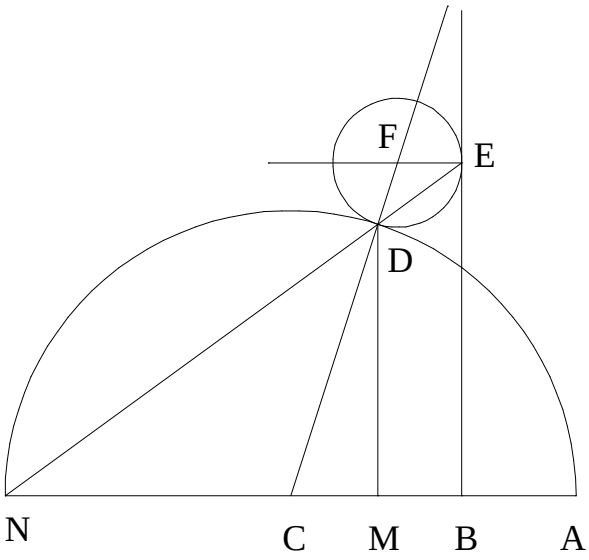
$AC := 3.125$ $AB := 4.25$ $AM := 5.34857$

$CD := AC$ $CN := AC$ $CM := AC - AM$ $DM := \sqrt{CD^2 - CM^2}$

$MN := CM + CN$ $BC := AC - AB$ $BN := CN + BC$ $DN := \sqrt{DM^2 + MN^2}$

$EN := \frac{DN \cdot BN}{MN}$ $ED := EN - DN$ $EF := \frac{AC \cdot ED}{DN}$ $EF = 3.80843$

The resultant equation in terms of the givens is $EF := \frac{AC \cdot (AM - AB)}{2 \cdot AC - AM}$ $EF = 3.80843$



Segment B.

Given AC, AB, DN, find EF.

$DN := 2.37359$

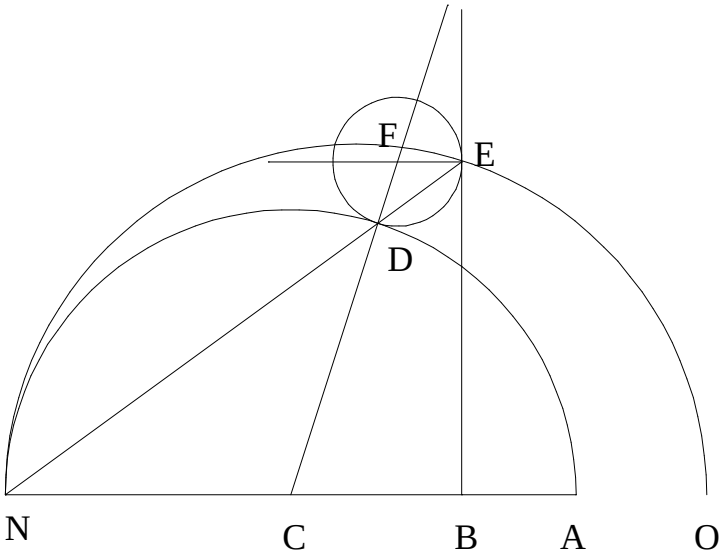
$AN := 2 \cdot AC$ $\quad MN := \frac{DN^2}{AN}$ $\quad CN := AC$ $\quad BC := AC - AB$ $\quad BN := CN + BC$

$EN := \frac{DN \cdot BN}{MN}$ $\quad ED := EN - DN$ $\quad EF := \frac{AC \cdot ED}{DN}$ $\quad EF = 3.80844$

The equation for this process is

$$EF := \frac{AC \cdot (4 \cdot AC^2 - 2 \cdot AC \cdot AB - DN^2)}{DN^2}$$

$EF = 3.80844$



Segment C.

Given AC, AB, BE, find EF.

$BE := 4.87173$

$AN := 2 \cdot AC$ $BN := AN - AB$ $EN := \sqrt{BE^2 + BN^2}$ $ON := \frac{EN^2}{BN}$

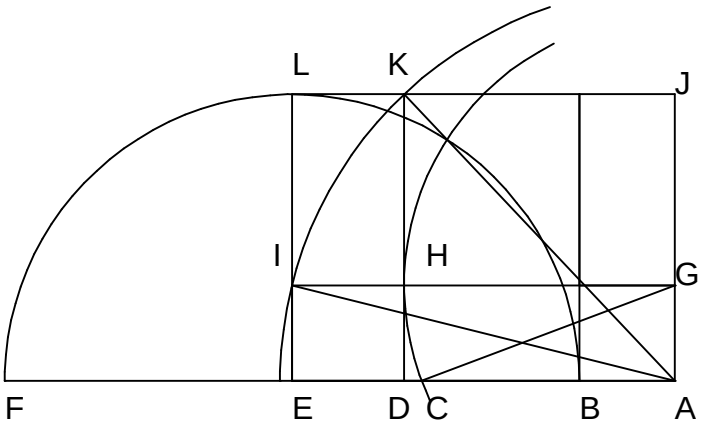
$DN := \frac{EN \cdot AN}{ON}$ $DE := EN - DN$ $EF := \frac{AC \cdot DE}{DN}$ $EF = 3.80844$

The equation for this process is $EF := \frac{BE^2 - 2 \cdot AC \cdot AB + AB^2}{2 \cdot (2 \cdot AC - AB)}$ $EF = 3.80844$

10_14_5C.MCD

Trivial Method: Square Root

Generalize the figure of 10_14_95.MCD



Starting at any point G, between A and J, the square root of AB·AF can always be projected to point C. Such a progression can be used on the cube root figure.

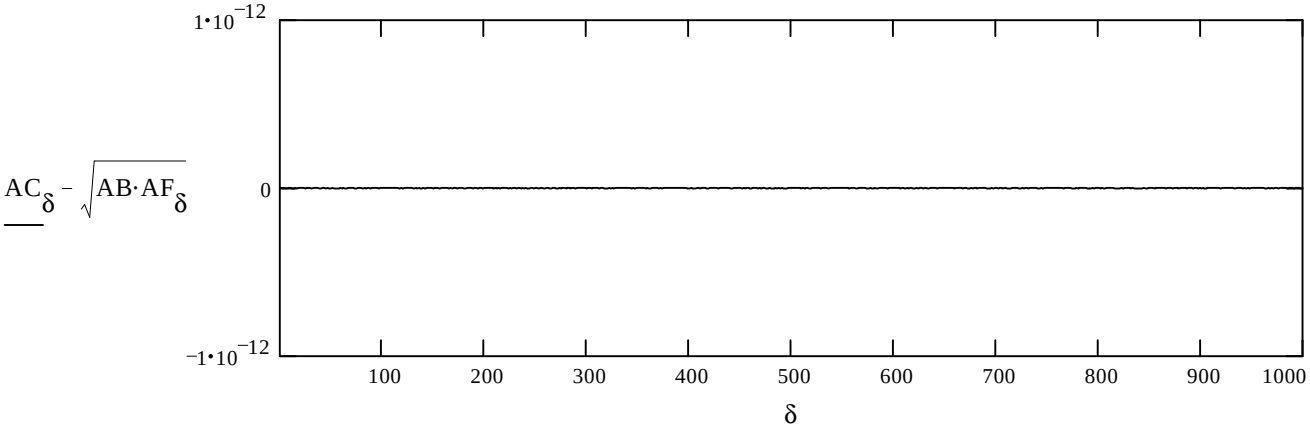
$\delta := 1 .. 1000 \quad AB := 10 \quad BF := 10 \quad BE := \frac{BF}{2}$

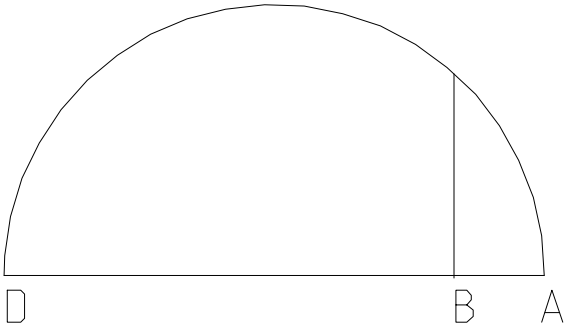
$AE := BE + AB \quad AJ := BE \quad AG_\delta := \frac{AJ}{\delta}$

$EI_\delta := AG_\delta \quad AI_\delta := \sqrt{(AE)^2 + (EI_\delta)^2}$

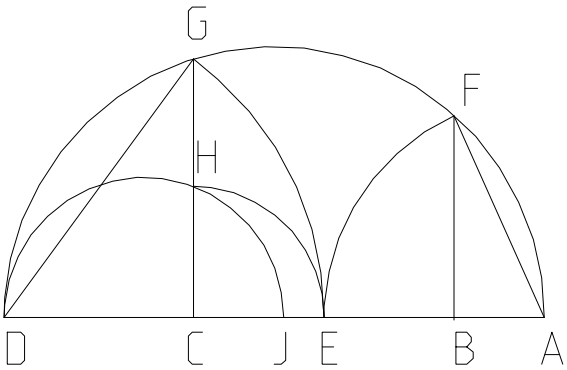
$AK_\delta := AI_\delta \quad DK := AJ \quad AD_\delta := \sqrt{(AK_\delta)^2 - DK^2} \quad GH_\delta := AD_\delta \quad CG_\delta := GH_\delta$

$AF_\delta := BF + AB \quad AC_\delta := \sqrt{(CG_\delta)^2 - (AG_\delta)^2}$





Given AB and BD divide BD such that $AB \cdot CD = \frac{BC^2}{4}$. And what is the reltionship of AC to AB and BD? Now the date on this file is not exact as I sketched this out on a piece of paper and forgot to date it.



$AB := 5 \quad BD := 1 \quad AD := AB + BD$

$BF := \sqrt{AB \cdot BD} \quad AE := \sqrt{AB^2 + BF^2}$

$DE := AD - AE \quad DG := DE \quad CD := \frac{DG^2}{AD}$

$CD = 0.046$

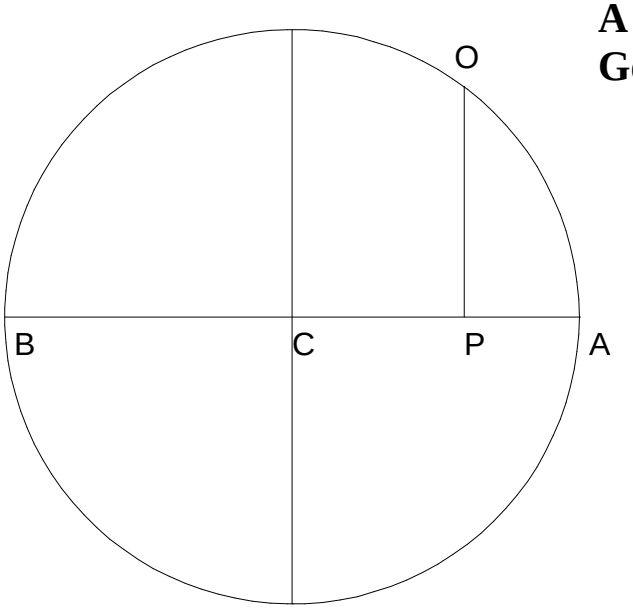
$BC := BD - CD \quad AB \cdot CD - \frac{BC^2}{4} = 0$

$CE := DE - CD \quad CH := CE \quad CJ := \frac{CH^2}{CD} \quad AB - CJ = 0$

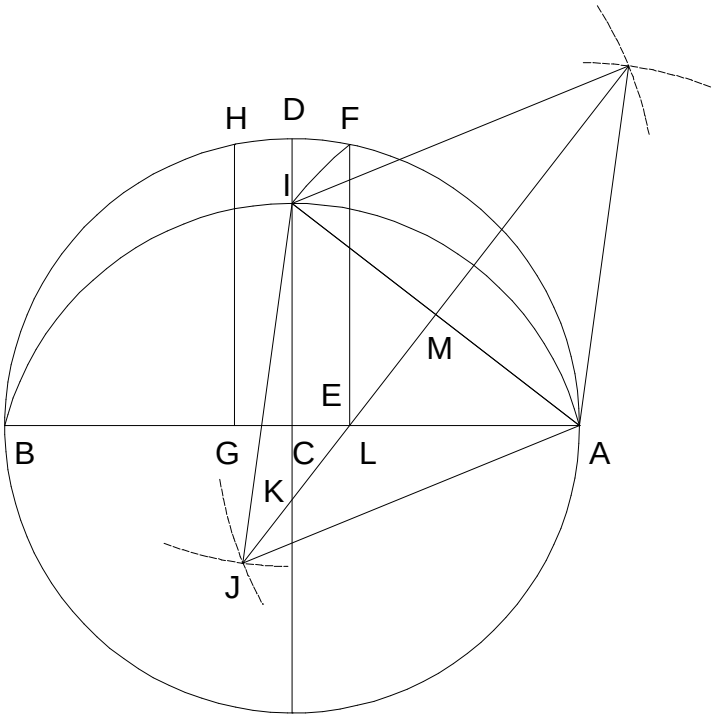
$AC := AB + BC \quad AC = 5.954$

$$AC - \left[2 \cdot \frac{AB^{\left(\frac{3}{2}\right)}}{\sqrt{AB + BD}} + 2 \cdot \frac{\sqrt{AB}}{\sqrt{AB + BD}} \cdot BD - AB \right] = 0$$

A Modification of a Square Root Figure. Gemini Roots



One of the square root figures displays a one to one ratio between what could be called the vertical segment OP and the root of the two horizontal segments AP and BP. With a slight modification however, one can demonstrate a many to one relationship between three base segments.



GL has a ratio to the root of $AL \cdot BG$. Developing the arc AIC from it will give a means of keeping that ratio.

BS = Base Segments, set at end of doc.

$$AB := 10 \qquad AC := \frac{AB}{2} \qquad EG := \frac{AB}{BS}$$

$$BC := AC \qquad CD := AC \qquad CE := \frac{EG}{2}$$

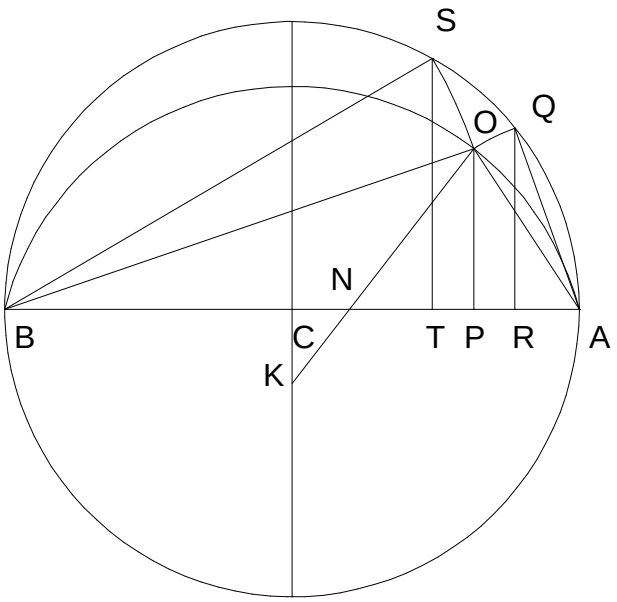
$$AE := AC - CE \qquad BE := BC + CE$$

$$EF := \sqrt{AE \cdot BE} \qquad AF := \sqrt{AE^2 + EF^2}$$

$$AI := AF \qquad AM := \frac{AI}{2} \qquad CI := \sqrt{AI^2 - AC^2}$$

$$AL := \frac{AI \cdot AM}{AC} \qquad CL := AC - AL$$

$$CK := \frac{AC \cdot CE}{CI} \qquad IK := CI + CK$$



$$\delta := 1 .. \Delta \qquad AN_\delta := \frac{AC}{\Delta}\cdot\delta$$

$$CN_\delta := AC - AN_\delta \qquad KO := IK$$

$$KN_\delta := \sqrt{CK^2 + \left(CN_\delta\right)^2} \qquad NO_\delta := KO - KN_\delta$$

$$NP_\delta := \frac{CN_\delta \cdot NO_\delta}{KN_\delta} \qquad AP_\delta := AC - CN_\delta - NP_\delta$$

$$OP_\delta := \frac{CK \cdot NO_\delta}{KN_\delta} \qquad AO_\delta := \sqrt{\left(AP_\delta\right)^2 + \left(OP_\delta\right)^2}$$

$$AQ_\delta := AO_\delta \qquad AR_\delta := \frac{\left(AQ_\delta\right)^2}{AB}$$

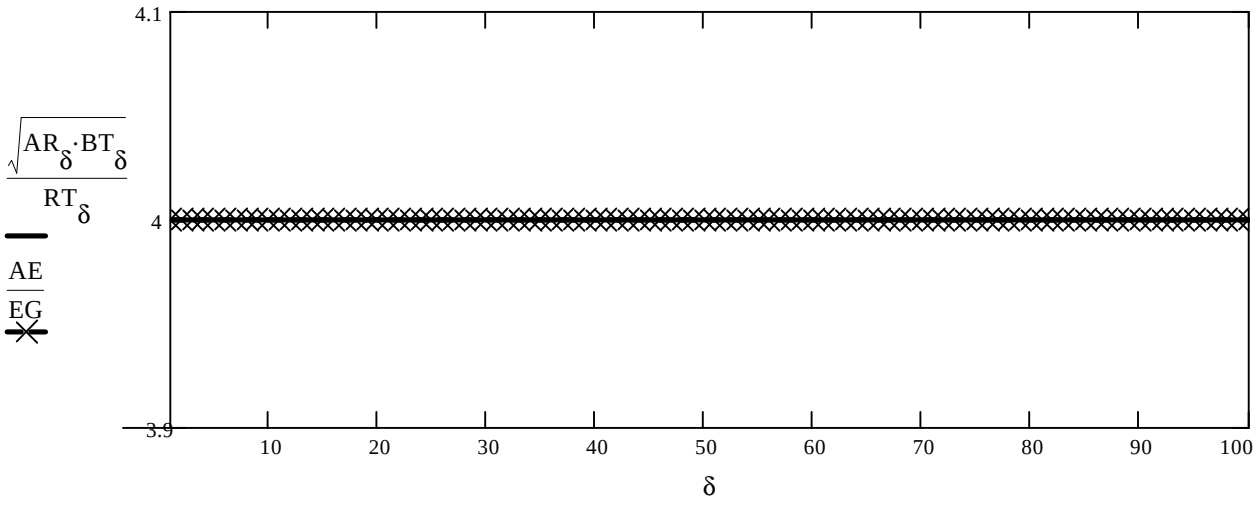
$$BP_\delta := BC + CN_\delta + NP_\delta$$

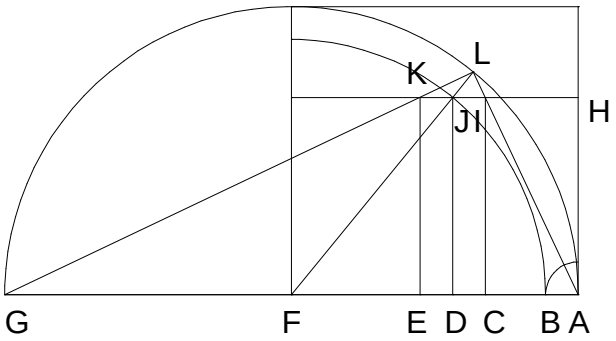
$$BO_\delta := \sqrt{\left(BP_\delta\right)^2 + \left(OP_\delta\right)^2} \qquad BS_\delta := BO_\delta \qquad BT_\delta := \frac{\left(BS_\delta\right)^2}{AB} \qquad AT_\delta := AB - BT_\delta \qquad RT_\delta := AT_\delta - AR_\delta$$

Set the number of Base Segments here and see if a constant relationship is expressed in the graph.

Set the number of drawings in Δ.

$BS \equiv 9$ $\Delta \equiv 100$





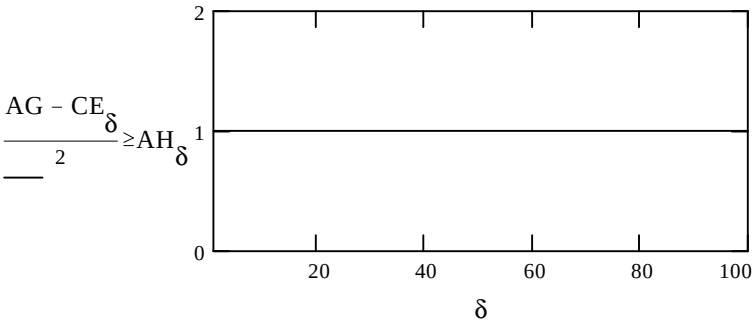
Short Method Gemini Roots.

Given AG, CE, AH, place CE so that CE:AH as CE:CI. Or more simply that CI $= \sqrt{AC_\delta \cdot EG_\delta} = AH$.

$\delta := 1 .. 100$ $\Delta := 8$

$AG := \Delta \cdot 2 + 1$ $CE_\delta := \frac{1}{\delta}$ $AH_\delta := \Delta \cdot CE_\delta$

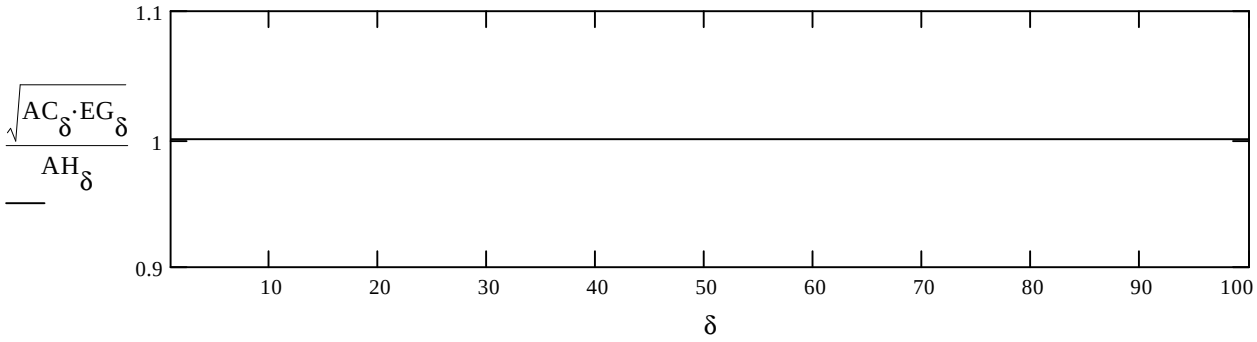
With the values given is the constuction possible? (1 for yes and 0 for no.)

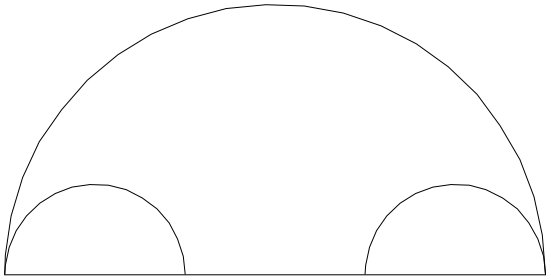


$DJ_\delta := AH_\delta$ $AB_\delta := \frac{CE_\delta}{2}$ $AF := \frac{AG}{2}$ $FG := AF$ $BF_\delta := AF - AB_\delta$ $FJ_\delta := BF_\delta$

$FD_\delta := \sqrt{(FJ_\delta)^2 - (DJ_\delta)^2}$ $DC_\delta := AB_\delta$ $DE_\delta := AB_\delta$ $AD_\delta := AF - FD_\delta$

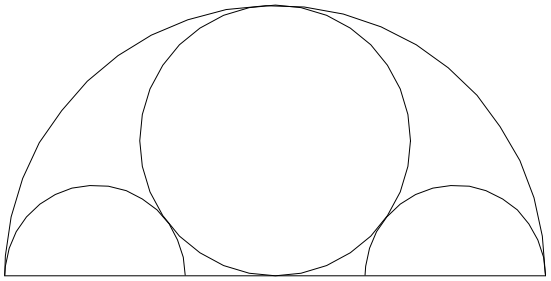
$AC_\delta := AD_\delta - DC_\delta$ $AE_\delta := AD_\delta + DE_\delta$ $EF_\delta := AF - AE_\delta$ $EG_\delta := EF_\delta + FG$

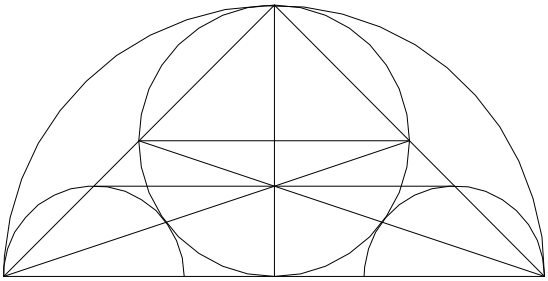


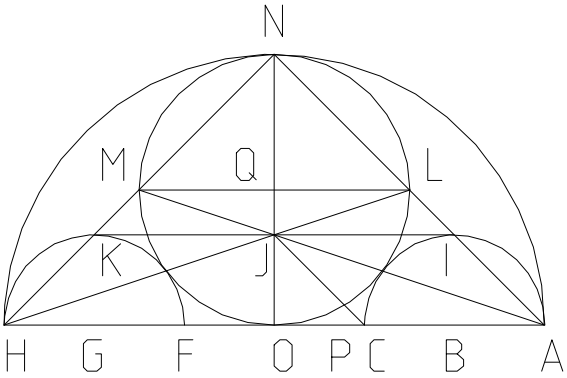


Method for Equals.

At the inner extremities of a great circle I have two equal smaller ones. Find the circle tangent to all three







$AH := 10 \quad AC := 3 \quad AO := \frac{AH}{2} \quad AB := \frac{AC}{2}$

$JO := AB \quad OP := JO \quad HO := AO$

$JP := \sqrt{JO^2 + OP^2} \quad HP := HO + OP$

$AL := \frac{JP \cdot AH}{HP} \quad NO := AO \quad AN := \sqrt{AO^2 + NO^2}$

$LN := AN - AL \quad LQ := \frac{AO \cdot LN}{AN} \quad LQ = 2.692$

Reducing LQ as an expression of the two givens, $LQ_F := \frac{AH \cdot (AH - AC)}{2 \cdot (AH + AC)} \quad LQ_F - LQ = 0$

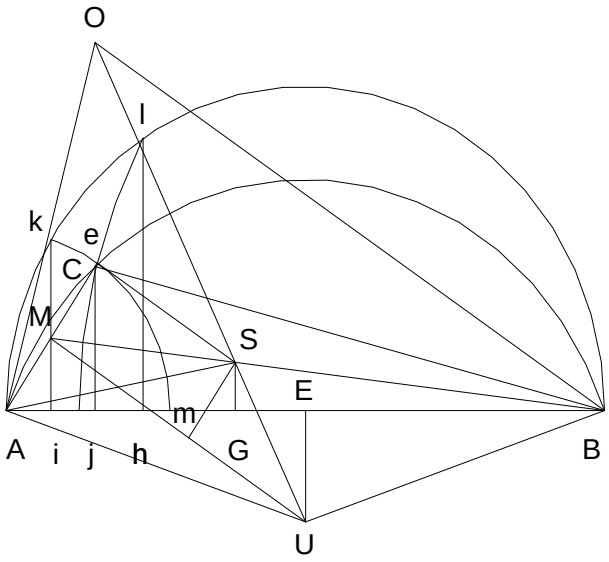
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2$ $AC := \begin{pmatrix} Side_1 \\ Side_2 \\ Side_3 \end{pmatrix}$ $BC := \begin{pmatrix} Side_2 \\ Side_3 \\ Side_1 \end{pmatrix}$ $AB := \begin{pmatrix} Side_3 \\ Side_1 \\ Side_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side_1 + Side_2 > Side_3)\cdot(Side_1 + Side_3 > Side_2)\cdot(Side_2 + Side_3 > Side_1)$



$AE_\delta := \frac{AB_\delta}{2}$ $Ak_\delta := AC_\delta$ $Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta}$ $Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$ $Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta$ $Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2}$ $BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta$ $Bg_\delta := \frac{BC_\delta}{2}$ $Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta$ $Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

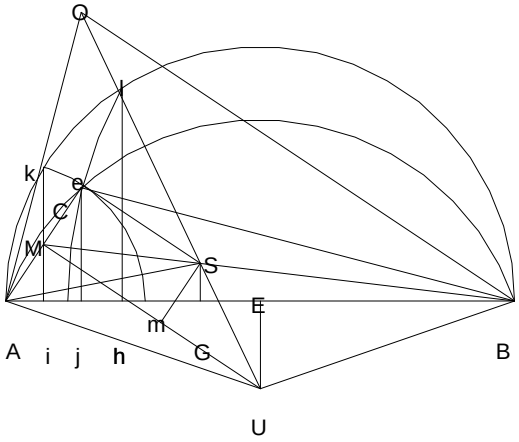
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2}$ $AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta}$ $BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2}$ $BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3}$ $BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta}$ $GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta$ $AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$



$MS_\delta := BM_\delta - BS_\delta$ $AU_\delta := BU_\delta$ $MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2}$ $Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

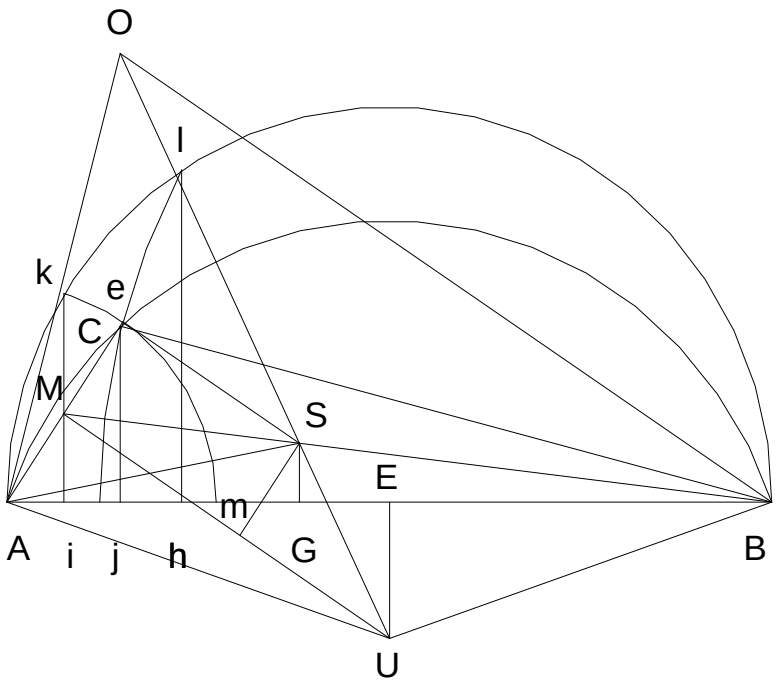
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

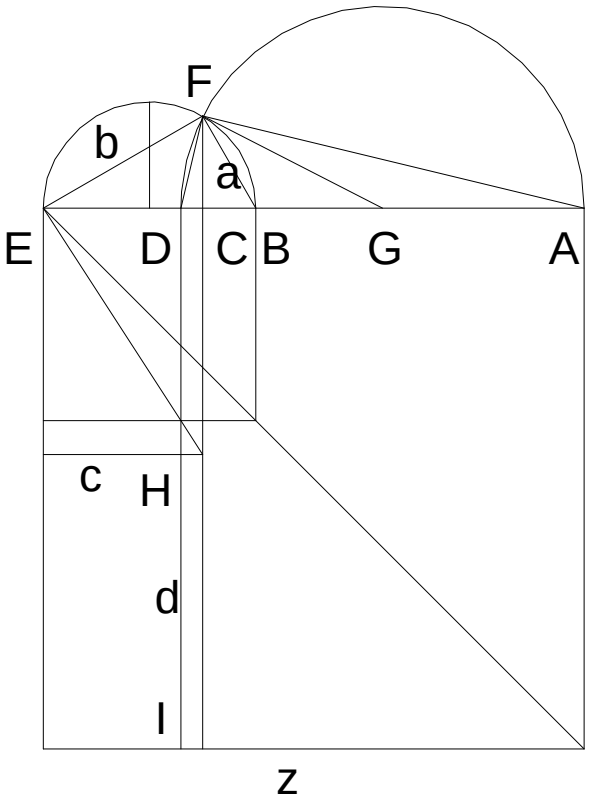


$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

0
1
1

SU_δ	UO_δ	AU_δ
14.15083	42.45249	16.70208
14.15083	42.45249	16.70208
14.15083	42.45249	16.70208

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.



12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a \qquad BE := \sqrt{a^2 + b^2} \qquad BC := \frac{a^2}{BE}$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2} \qquad CG := \sqrt{FG^2 - CF^2}$

$AG := FG \qquad AC := AG + CG$

$BG := CG - BC \qquad DG := FG$

$BD := DG - BG \qquad AB := AG + BG$

$AE := AB + BE \qquad DE := BE - BD$

$DH := \frac{b^2}{DE} \qquad DI := AE \qquad HI := DI - DH$

$z := AE \qquad z = 12.622$

$c := DE \qquad c = 0.622$

$d := HI \qquad d = 6.186$

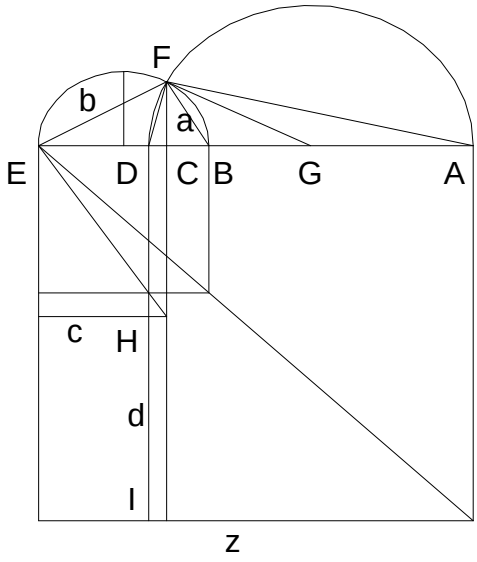
$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

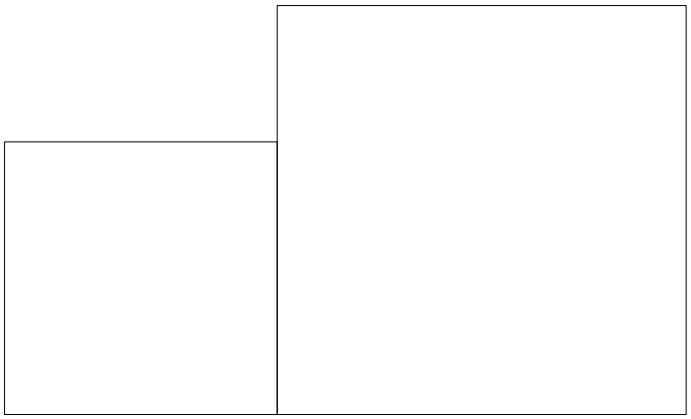
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$

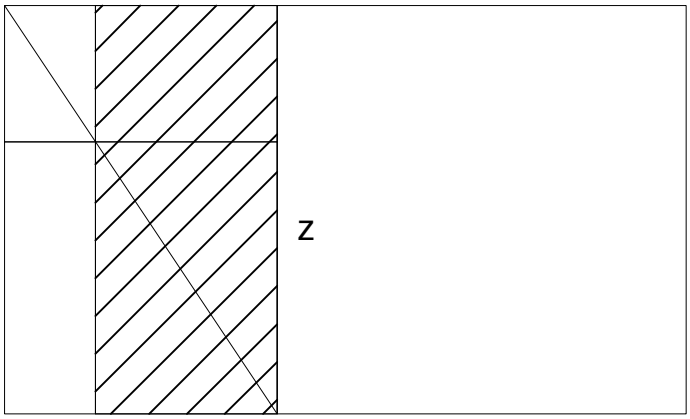


12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

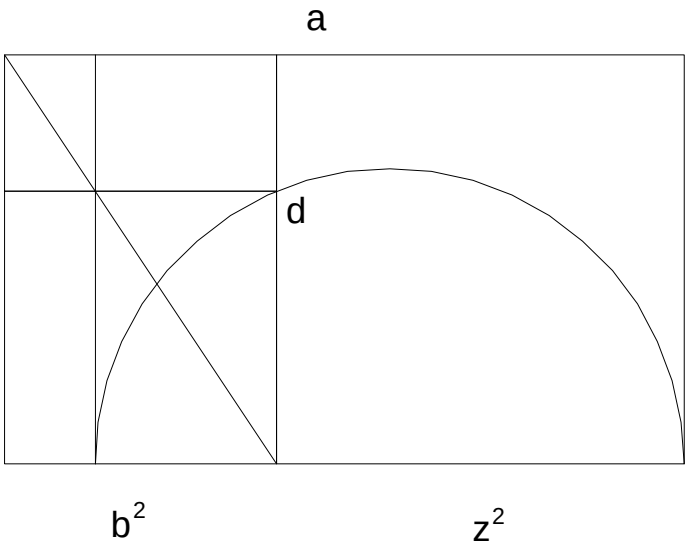


$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.



$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

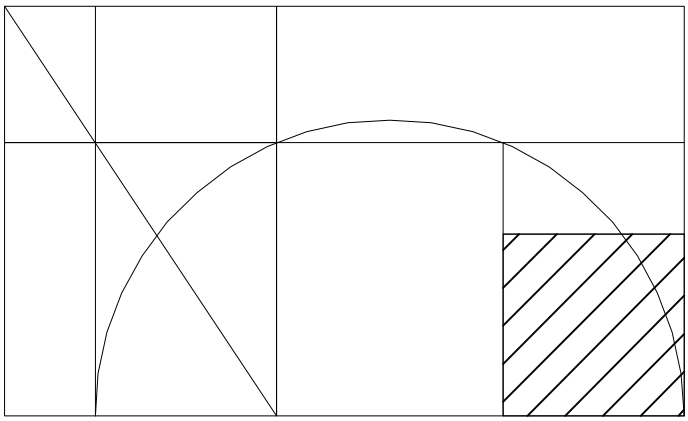
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

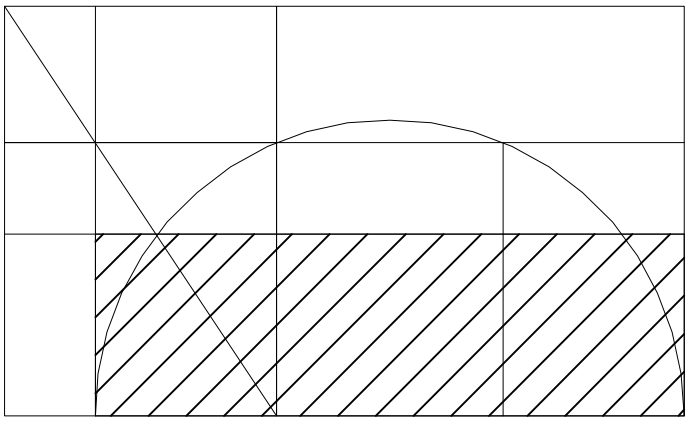
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

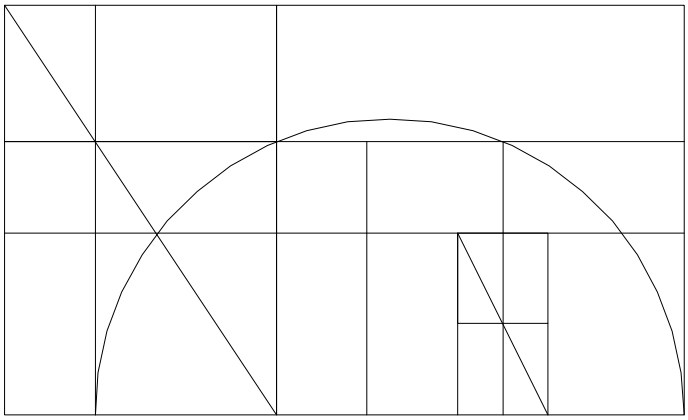
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



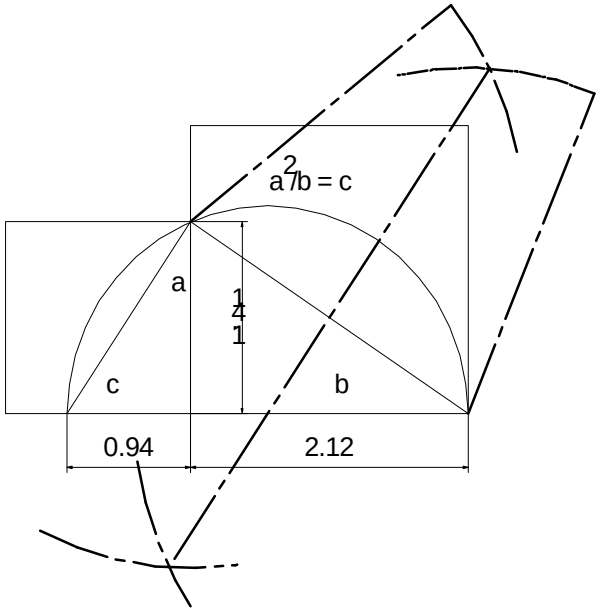
b^2 z^2



b^2 z^2

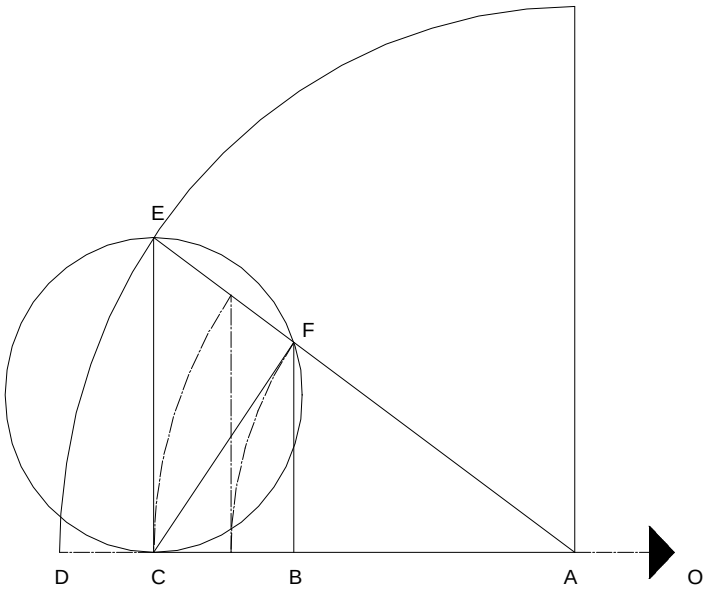


b^2 z^2



The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

Just for fun.



$N := 1.3$

$AD := 42 \qquad AC := \frac{AD}{N} \qquad CO := AD + AC$

$CD := AD - AC \qquad CE := \sqrt{CD \cdot CO}$

$AF := \frac{AC^2}{AD} \qquad AB := \frac{AF \cdot AC}{AD}$

$AC^3 - \left(AD^2 \cdot AB\right) = 0$

$AB = 19.117$

$AC - \left(AD^2 \cdot AB\right)^{\frac{1}{3}} = 0$

The ratio of BC to CE is common to another cube root expression, which?

Pascal's triangle with exponential division.

$AB := 3 \qquad AD := 5$

$BD := AD - AB \qquad AC := \sqrt{AB \cdot AD}$

$CD := AD - AC \qquad BC := BD - CD$

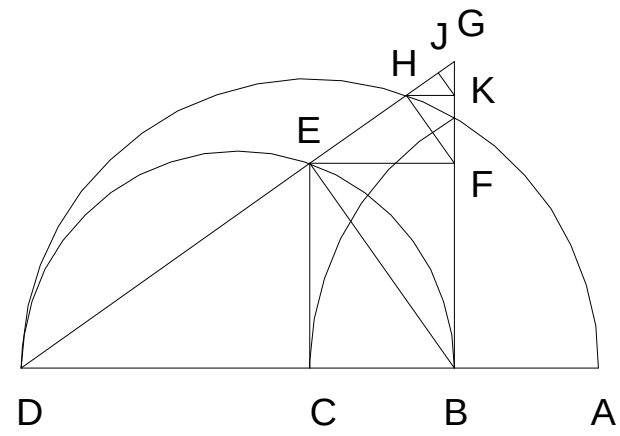
$CD := BD - BC \qquad CE := \sqrt{BC \cdot CD}$

$BE := \sqrt{BC^2 + CE^2} \qquad DE := \sqrt{CD^2 + CE^2}$

$BG := \frac{BD \cdot BE}{DE} \qquad FG := \frac{BC \cdot BG}{BD}$

$EG := \frac{BE \cdot BG}{BD} \qquad DG := DE + EG$

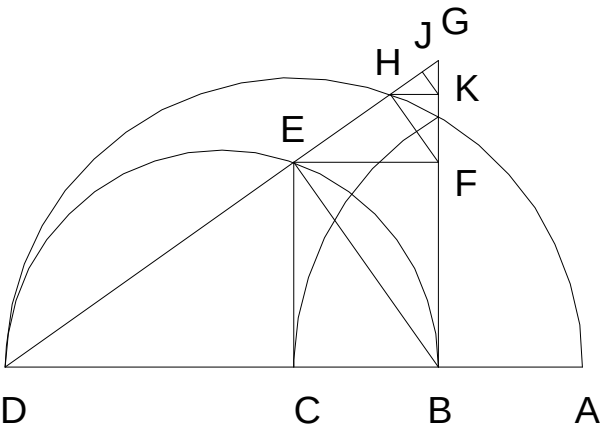
$GH := \frac{BC \cdot EG}{BD} \qquad GJ := \frac{BC \cdot GH}{BD}$



$\frac{BD}{BC} = 2.291 \qquad \frac{DG}{GH} = 5.249 \qquad \frac{DG}{GJ} = 12.025$

$N := 1 .. 3 \qquad \frac{\left(\sqrt{AD} + \sqrt{AB}\right)^N}{\sqrt{AB^N}}$

2.291
5.249
12.025



$\frac{A + B}{A}$

$\sqrt{AD} + \sqrt{AB}$

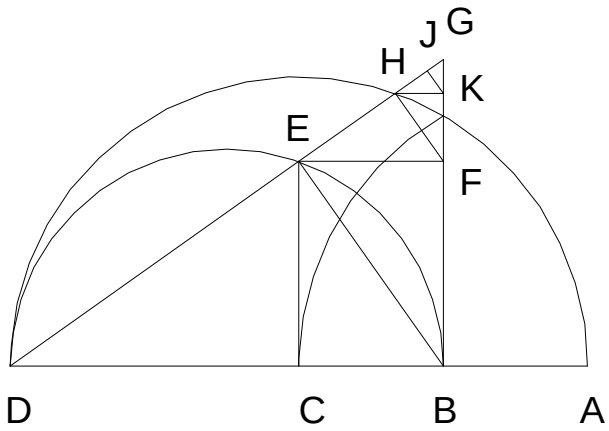
$\frac{A^2 + 2AB + B^2}{A^2}$

$\left(\sqrt{AD}\right)^2 + 2 \cdot \sqrt{AB} \cdot \sqrt{AD} + \left(\sqrt{AB}\right)^2$

$\frac{A^3 + 3A^2 \cdot B + 3A \cdot B^2 + B^3}{A^3}$

$\left(\sqrt{AD}\right)^3 + 3 \cdot AD \cdot \sqrt{AB} + 3 \cdot \sqrt{AD} \cdot AB + \left(\sqrt{AB}\right)^3$

Dividing an exponentiated integer by an exponentiated integer of the same power, straight edge and compass construction. Followed by who knows what!



$CD := 1.5 \qquad BC := .75 \qquad BD := BC + CD$

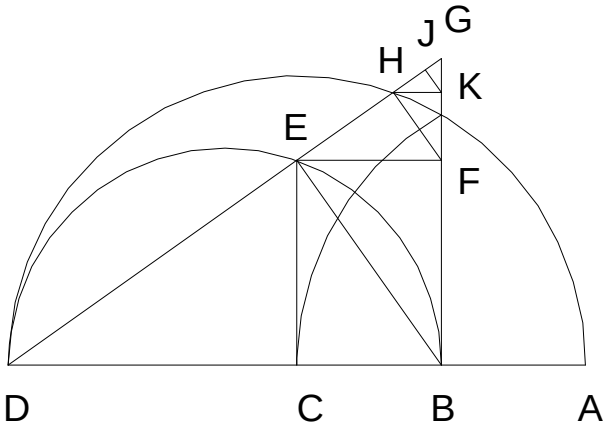
$CE := \sqrt{BC \cdot CD} \qquad BE := \sqrt{BC^2 + CE^2}$

$EF := BC \qquad FG := \frac{BC \cdot EF}{CE} \qquad FG = 0.53$

$DE := \sqrt{CD^2 + CE^2} \qquad EG := \frac{BD \cdot EF}{DE}$

$GH := \frac{BC \cdot EF}{DE} \qquad DG := DE + EG$

$JK := \frac{FG \cdot GH}{EG} \qquad GJ := \frac{BC \cdot GH}{BD} \qquad FH := \frac{CE \cdot EG}{BD} \qquad \frac{BD}{BC} = 3 \qquad \frac{DG}{GH} = 9 \qquad \frac{DG}{GJ} = 27$



$n := 1..3$

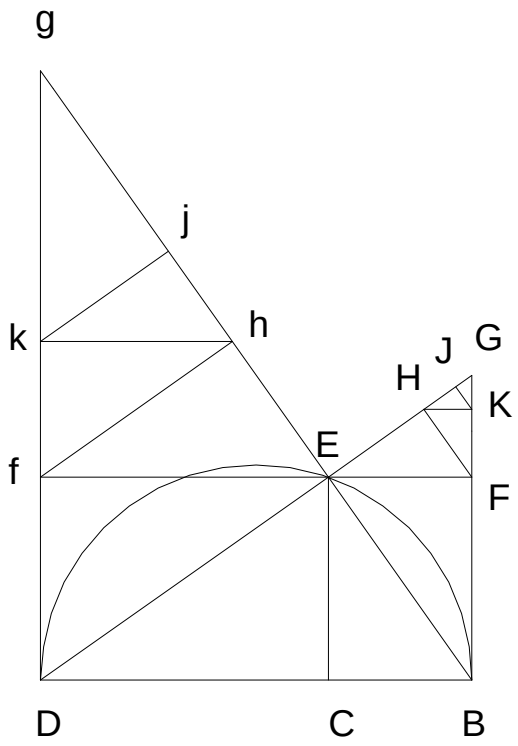
$\frac{a^n}{b^n}$ $\frac{BD^n}{BC^n}$

3
9
27

$BC + CD = 2.25$ $\sqrt{DG \cdot DE} = 2.25$

$\frac{\sqrt{BC + CD} \cdot BC}{\sqrt{CD}} = 0.919$ $\sqrt{DG \cdot GH} = 0.919$

$\frac{\left(\sqrt{BC}\right)^3}{\sqrt{CD}} = 0.53$ $\sqrt{DG \cdot GJ} = 0.53$



$Ef := CD$ $Eg := \frac{BD \cdot Ef}{BE}$ $fh := \frac{CE \cdot Ef}{BE}$

$hk := \frac{CE \cdot fh}{BE}$ $hj := \frac{BC \cdot hk}{BE}$ $Eg := \frac{BD \cdot Ef}{BE}$

$Bg := BE + Eg$ $hg := \frac{CD \cdot Ef}{BE}$ $jg := \frac{CD \cdot hk}{BE}$

$\frac{BD}{CD} = 1.5$ $\frac{Bg}{hg} = 2.25$ $\frac{Bg}{jg} = 3.375$ $\left(\frac{BD}{CD}\right)^n$

1.5
2.25
3.375

$\frac{Bg}{EG} = 4.243$ $\frac{Bg}{GH} = 12.728$ $\frac{Bg}{GJ} = 38.184$ $\sqrt{CD} \cdot \frac{(BC + CD)^n}{\left(\sqrt{BC}\right)^{2 \cdot n + 1}}$

4.243
12.728
38.184

$n := 1 .. 4$

$\sqrt{Bg \cdot GJ} = 0.631$

$\sqrt{Bg \cdot GH} = 1.092$

$\sqrt{Bg \cdot EG} = 1.892$

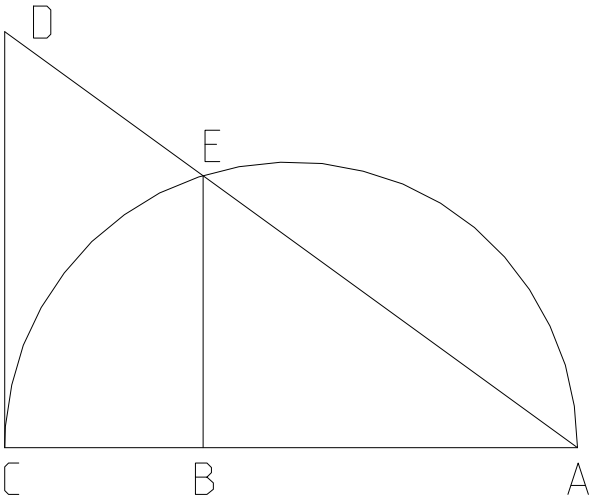
$\sqrt{Bg \cdot DG} = 3.277$

$\frac{BC^{\left(\frac{7}{4}\right)}}{CD^{\left(\frac{1}{4}\right)} \cdot \sqrt{BC + CD}} \cdot \frac{\left(\sqrt{BC + CD}\right)^n}{\left(\sqrt{BC}\right)^n}$

0.631
1.092
1.892
3.277

$\frac{BC^{\left(\frac{7}{4} - \frac{1}{2} \cdot n\right)}}{CD^{\left(\frac{1}{4}\right)}} \cdot (BC + CD)^{\left(\frac{-1}{2} + \frac{1}{2} \cdot n\right)}$

0.631
1.092
1.892
3.277



Given AC and CD find BC when it is equal to $\frac{AC \cdot CD^2}{AC^2 + CD^2}$.

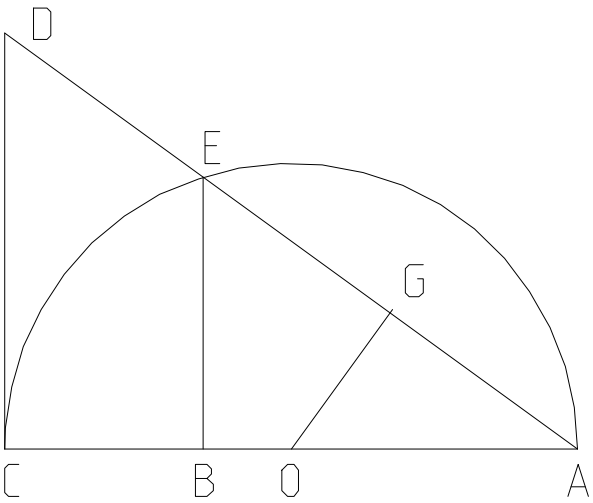
$AC := 15 \quad CD := 5 \quad AO := \frac{AC}{2}$

$AD := \sqrt{AC^2 + CD^2}$

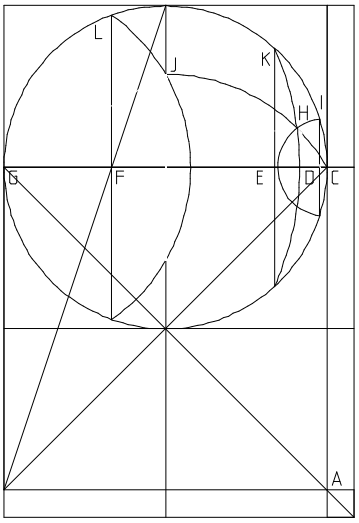
$AG := \frac{AC \cdot AO}{AD} \quad AE := 2 \cdot AG$

$AB := \frac{AC \cdot AE}{AD} \quad BC := AC - AB \quad BC = 1.5$

$BC := \frac{AC \cdot CD^2}{\left(AC^2 + CD^2\right)} \quad BC = 1.5$



A cube divided by the sum of two squares.



One Circle

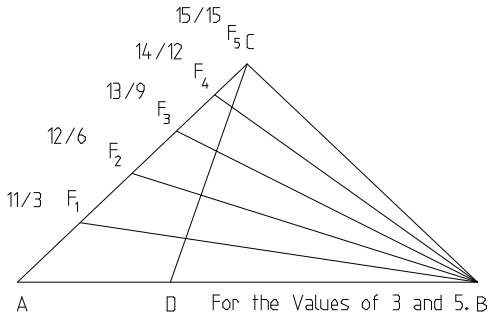
By John Clark

One Square

One Line

And the Delian Quest

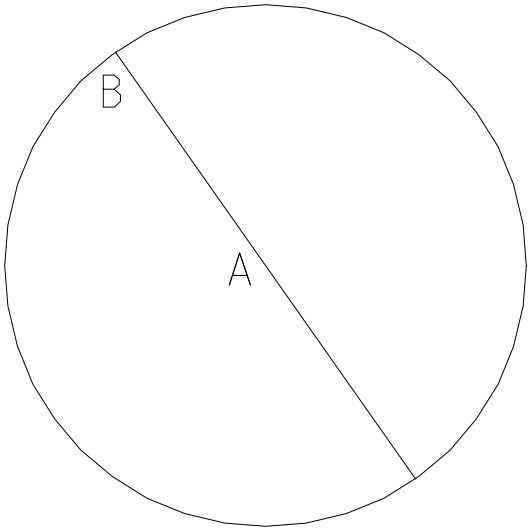
1996



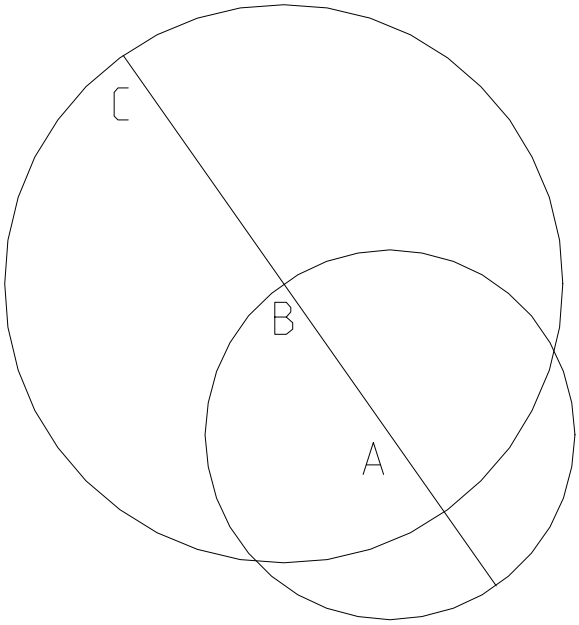
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

The Archamedian Paper Trisector- Without the Numbers.

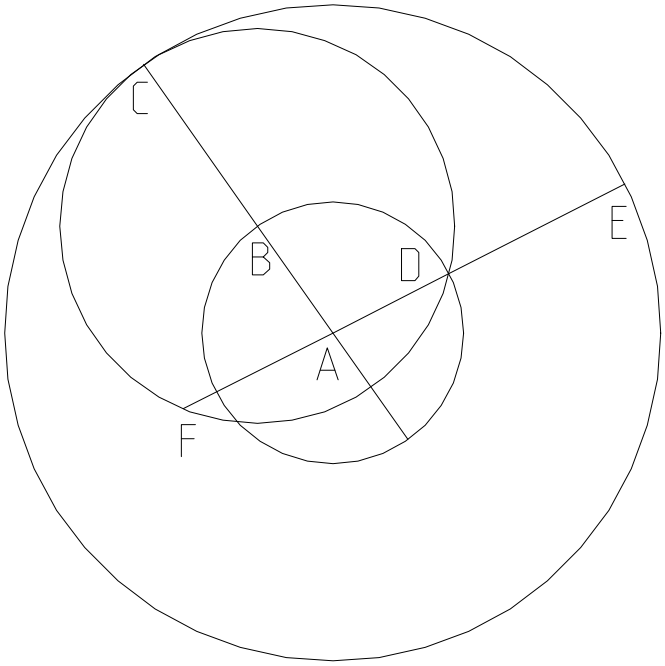
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.



Given any circle AB

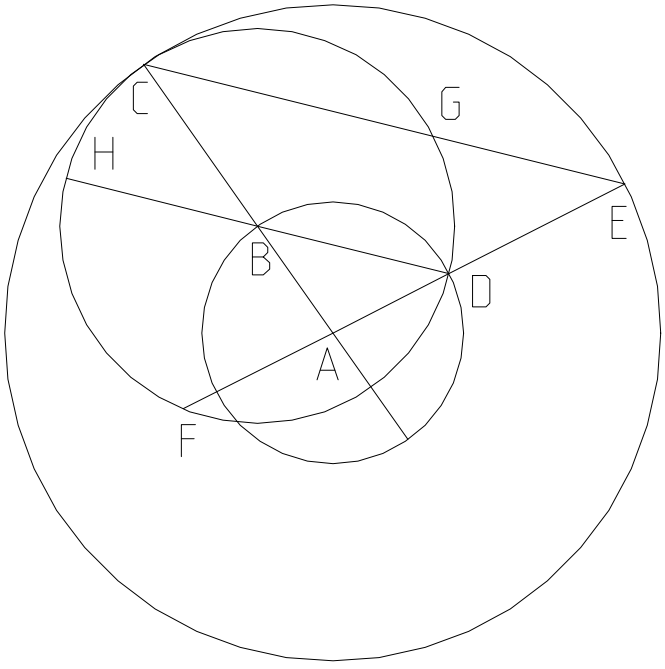


Given any circle BC such that BC≤ 2AB.

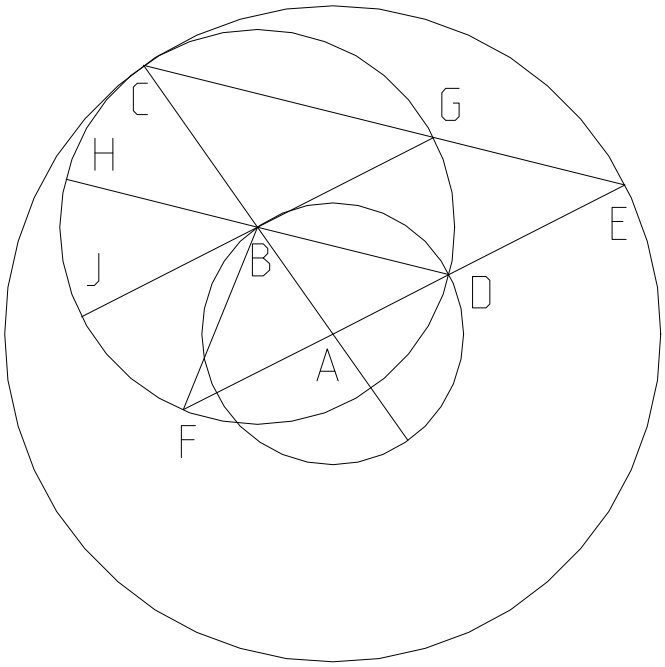


Construct AE such that AE = AC.

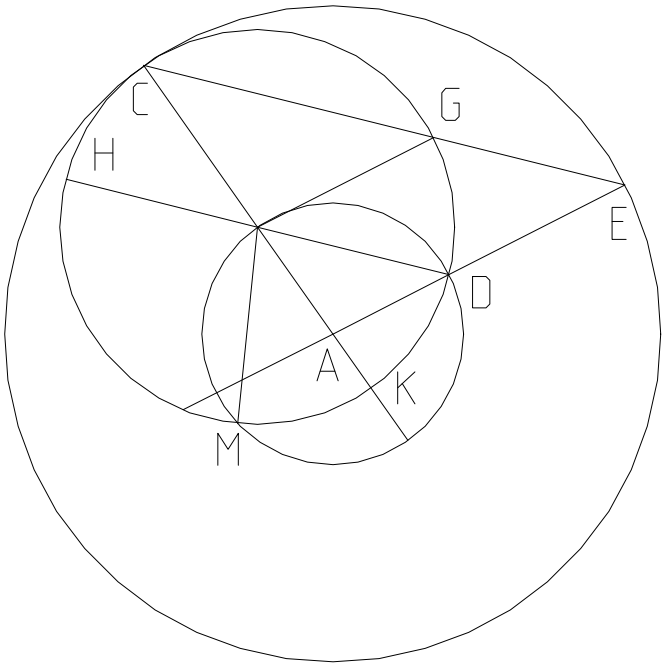
Since AC = AB+ BC and AD = AB, DE = BC.



Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



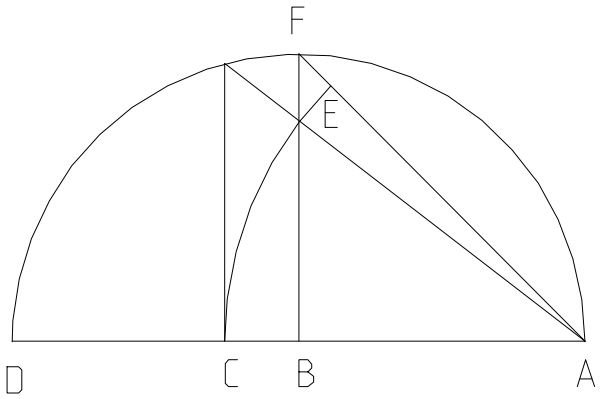
Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.



By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

A rusty Compass construction for the duplication of the cube.

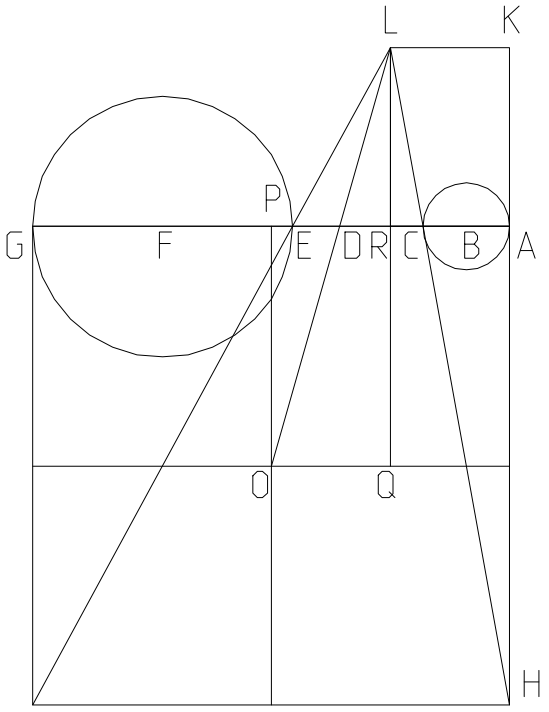


$AD := 2 \quad AB := \frac{AD}{2} \quad AF := \sqrt{2 \cdot AB^2} \quad AE := \frac{AF}{9} \cdot 8$

$AC := AE \quad AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26 \qquad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.



Alternate Method, Power Line.

Given AB, EF, BF, find the power line intersection on BF. Looking back to 94, it seems I never derived a formula for it either.

$AB := R_1 \quad BF := D \quad EF := R_2$

$BC := AB \quad AC := AB + BC \quad FG := EF$

$EG := EF + FG \quad AG := AB + BF + FG \quad AH := AG$

$GJ := AG \quad AP := \frac{AG}{2} \quad CE := BF - (BC + EF)$

$LR := \frac{AH \cdot CE}{AC + EG} \quad PO := AP \quad AK := LR$

$KL := \frac{AC \cdot (AH + AK)}{AH} \quad AR := KL$

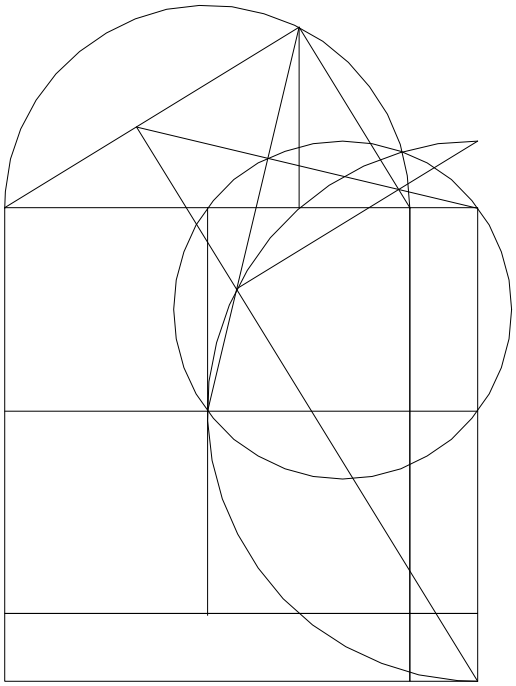
$PR := AP - AR \quad OQ := PR \quad QR := PO$

$LQ := QR + LR \quad DR := \frac{OQ \cdot LR}{LQ} \quad AD := AR + DR$

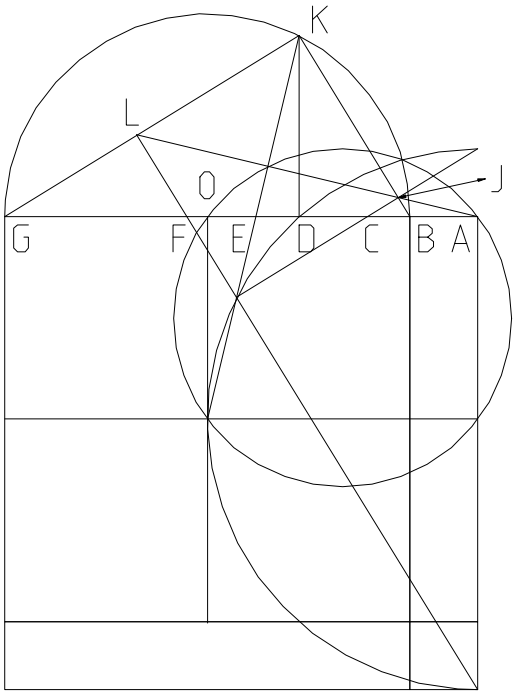
$AD = 25.333$ Plug values in below.

$R_1 \equiv 9 \qquad R_2 \equiv 1 \qquad D \equiv 30$

$$\frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D} = 25.333$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.



$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

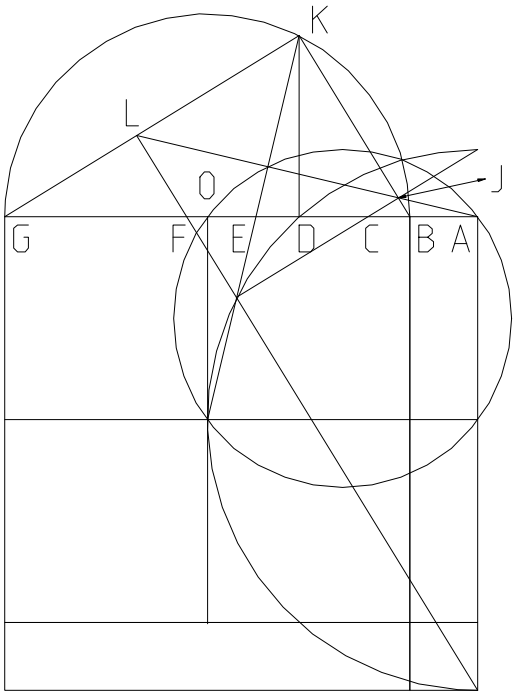
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



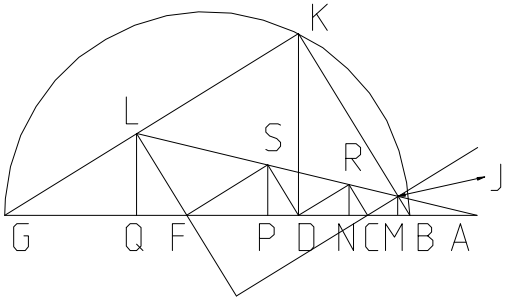
$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$



$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$$

$$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

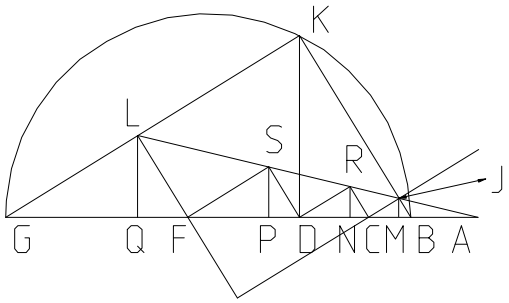
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$

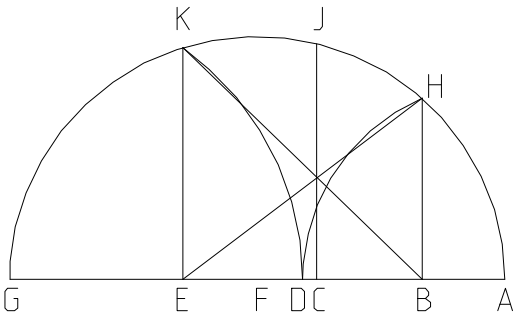


$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

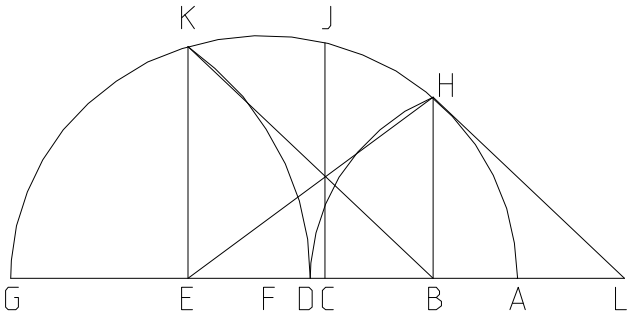
$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$



If the figure was drawn differently, XC would be $\sqrt{XB \cdot XE}$, irregardless of how XB and XE were placed, however that would require part of the figure that is not given here.



$$AG := N \quad AB := \frac{AG}{N}$$

$$BG := AG - AB \quad BH := \sqrt{AB \cdot BG}$$

$$AH := \sqrt{AB^2 + BH^2} \quad AD := AH \quad DG := AG - AD$$

$$GK := DG \quad GE := \frac{GK^2}{AG} \quad AE := AG - GE$$

$$BE := AE - AB$$

$$EG := AG - AE \quad EK := \sqrt{AE \cdot EG} \quad BL := \frac{BE \cdot BH}{EK}$$

$$EL := BE + BL \quad BC := \frac{BL \cdot BE}{EL} \quad AC := AB + BC$$

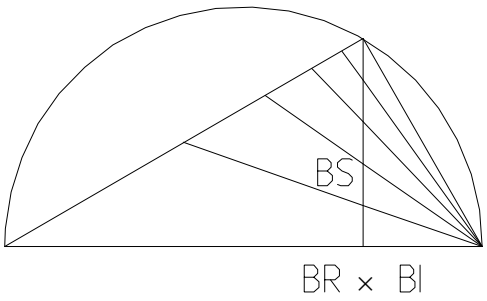
$N \equiv 4$

Make N any number and watch the equations, then make it equal to 1 and see what happens. Now this is strange work, for the formula is an identity with AC, so what happens at 1? This is an example of Binary contradiction.

$AC = 2$

$$\frac{\left[\begin{array}{l} AB^{\left(\frac{5}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{7}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + 2 \cdot AB \cdot \sqrt{BG} \cdot (AB + BG)^{\left(\frac{3}{2}\right)} \ldots \\ + - AB^{\left(\frac{5}{2}\right)} \cdot \sqrt{BG} - AB^{\left(\frac{3}{2}\right)} \cdot BG^{\left(\frac{3}{2}\right)} \end{array}\right]}{\left[\begin{array}{l} AB^{\left(\frac{1}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{3}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + AB^{\left(\frac{3}{2}\right)} \cdot \sqrt{BG} + \sqrt{AB} \cdot BG^{\left(\frac{3}{2}\right)} \end{array}\right]} = 2$$

Pyramid of Ratios, Moving the Point

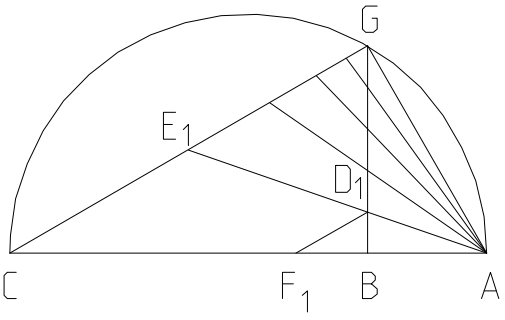


BR = Base Ratio, BS = Bisector Segments, BI = Base Index.

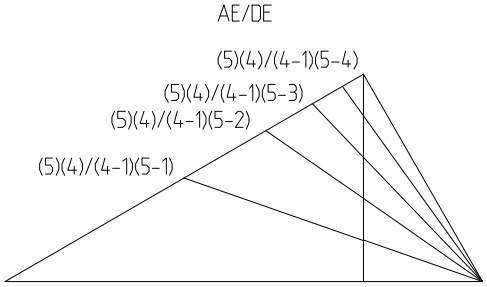
$BR \equiv 4 \quad BS \equiv 5 \quad BI := 2 \quad AC := BR \quad AB := \frac{AC}{BR} \cdot BI$

$BC := AC - AB \quad BG := \sqrt{AB \cdot BC} \quad \delta := 1 .. BS - 1$

$BD_\delta := \frac{BG}{BS} \cdot \delta \quad BF_\delta := \frac{BC \cdot BD_\delta}{BG} \quad AD_\delta := \sqrt{AB^2 + (BD_\delta)^2}$



$AE_\delta := \frac{AD_\delta \cdot AC}{AB + BF_\delta} \quad DE_\delta := AE_\delta - AD_\delta$

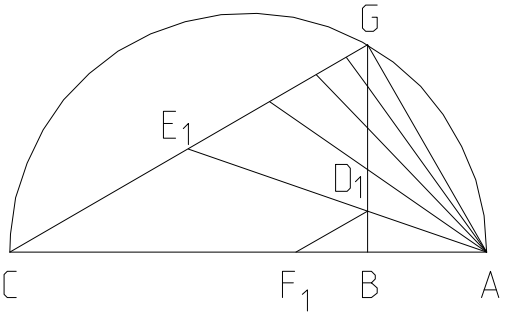


$\frac{AE_\delta}{DE_\delta}$

2.5
3.333
5
10

$\frac{BS \cdot BR}{(BR - BI) \cdot (BS - \delta)}$

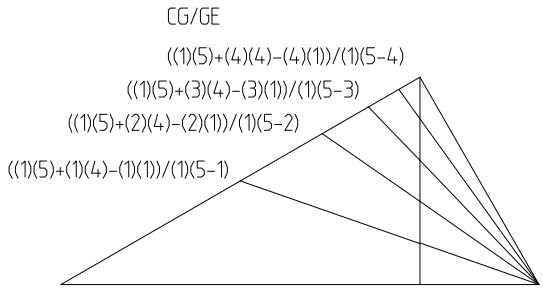
2.5
3.333
5
10



$AF_\delta := AB + BF_\delta \quad EJ_\delta := \frac{BD_\delta \cdot AC}{AF_\delta}$

$AJ_\delta := \sqrt{(AE_\delta)^2 - (EJ_\delta)^2} \quad CJ_\delta := AC - AJ_\delta$

$CE_\delta := \sqrt{(CJ_\delta)^2 + (EJ_\delta)^2} \quad CG := \sqrt{BG^2 + BC^2}$



$GE_\delta := CG - CE_\delta$

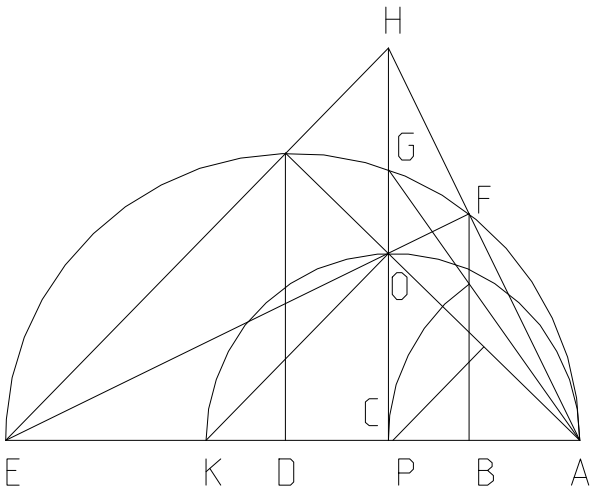
$\frac{CG}{GE_\delta}$

1.5
2.333
4
9

$\frac{(BI \cdot BS + \delta \cdot BR - \delta \cdot BI)}{(BI \cdot (BS - \delta))}$

1.5
2.333
4
9

What is AD. What is BD to $\sqrt{AB \cdot DE}$?



$AE := 5.5 \quad AB := 1.05 \quad BE := AE - AB$

$AC := \left(AB^2 \cdot AE\right)^{\frac{1}{3}} \quad CE := AE - AC$

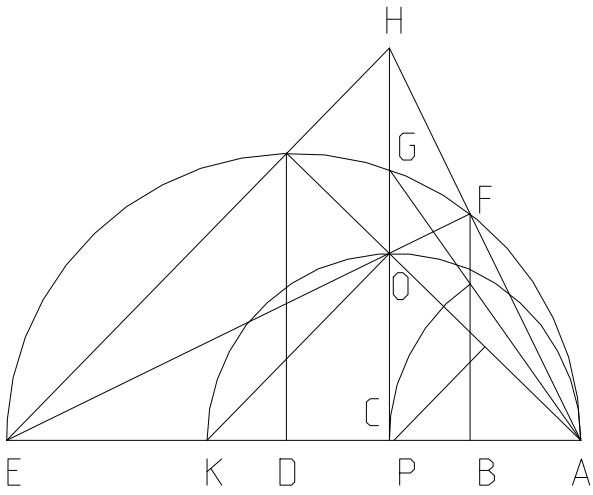
$BF := \sqrt{AB \cdot BE} \quad CO := \frac{BF \cdot CE}{BE} \quad AO := \sqrt{AC^2 + CO^2}$

$AP := \frac{1}{2} \cdot \frac{AO^2}{AC} \quad AK := 2 \cdot AP \quad AD := \frac{AC \cdot AE}{AK}$

$DE := AE - AD \quad BD := AD - AB$

$AD = 2.807$

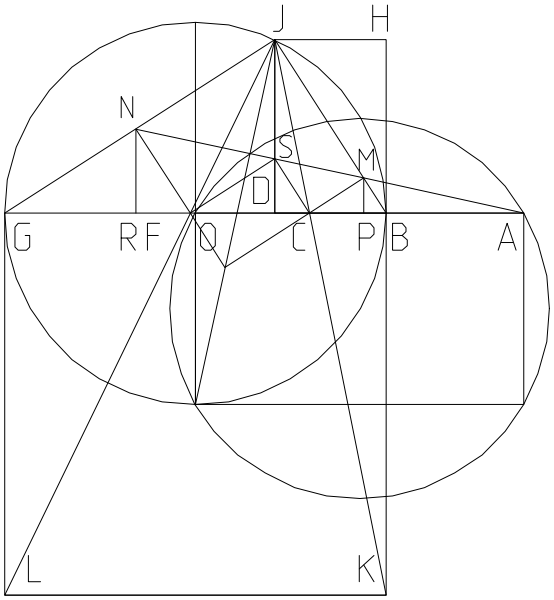
$$\frac{AE^{\frac{3}{3}} \cdot AB^{\frac{1}{3}} + AE^{\frac{2}{3}} \cdot AB^{\frac{2}{3}} + AE^{\frac{1}{3}} \cdot AB^{\frac{3}{3}}}{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}} = 2.807$$



$$\frac{\sqrt{AB \cdot DE}}{BD} = 0.957$$

$$\frac{AB^{\frac{1}{6}} \cdot AE^{\frac{1}{6}} \cdot \sqrt{AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} + AE - AE^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} \cdot \sqrt{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}}}{AE + AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} - 2 \cdot AB} = 0.957$$

The figure cuts the base in Cube Roots and provides some interesting ratios.



$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

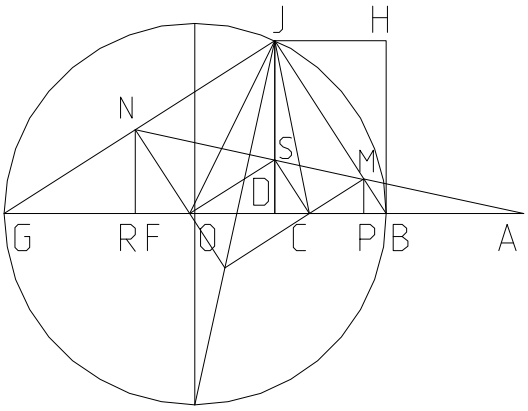
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$



$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

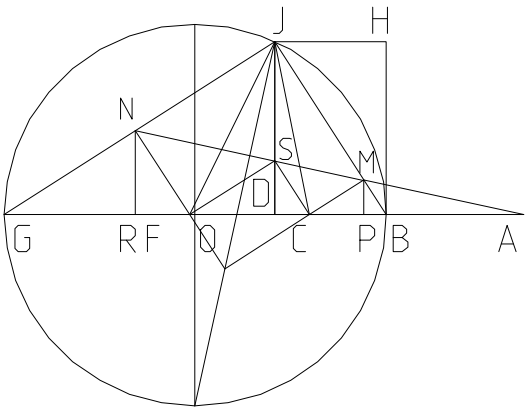
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

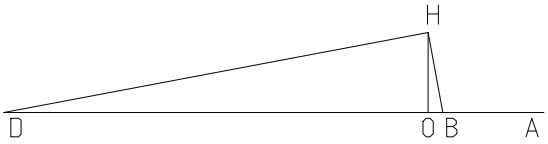
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$



$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

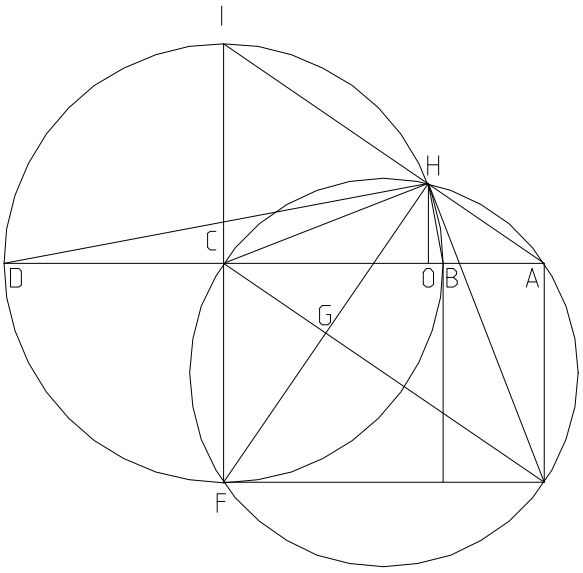
$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$



Given AD and AB on AD, place a right triangle on BD as base such that the opposite sides are in the ratio of AB to AD.

$BD := 8 \quad AB := 2 \quad AD := AB + BD$

$BC := \frac{BD}{2} \quad AC := AB + BC \quad CF := BC$



$CI := BC \quad CH := BC \quad AE := BC$

$CE := \sqrt{AC^2 + AE^2} \quad CG := \frac{CH^2}{CE}$

$GH := \sqrt{CH^2 - CG^2} \quad FH := 2 \cdot GH \quad FI := CF + CI$

$HI := \sqrt{FI^2 - FH^2} \quad AI := \sqrt{AC^2 + CI^2}$

$AH := AI - HI \quad AO := \frac{AC \cdot AH}{AI} \quad HO := \frac{CI \cdot AO}{AC}$

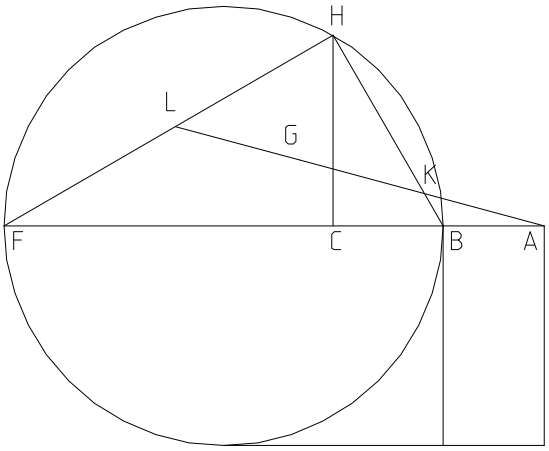
$BO := AO - AB \quad DO := BD - BO$

$DH := \sqrt{DO^2 + HO^2} \quad BH := \sqrt{BO^2 + HO^2}$

$$\frac{DH}{BH} = 5 \qquad \frac{AD}{AB} = 5$$

Given a straight edge and compass, AB and BD find the sum of six cubes divided by the sum of five squares.

$$\frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BD + AB \cdot BD^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BD + BD^2} = 2.308 \qquad AO = 2.308$$



Given AF and AB on AF and a right triangle on BF divide the sides of the triangle such that a section on one side is to the other as AB is to AF.

Now it can be realized that there are stipulations as to possible placements of the given triangle.

$AB := 3 \quad BF := 10 \quad BC := 1$

$AC := AB + BC \quad BE := \frac{BF}{2} \quad AE := AB + BE$

$DOABLE := \frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BF + AB \cdot BF^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BF + BF^2} \leq AC < AE$

$DOABLE = 1$

$AF := AB + BF$

$CF := BF - BC \quad CH := \sqrt{BC \cdot CF} \quad EJ := BE$

$CE := BE - BC \quad JH := \sqrt{CE^2 + (CH + EJ)^2}$

$DE := \frac{CE \cdot EJ}{EJ + CH} \quad AD := AE - DE$

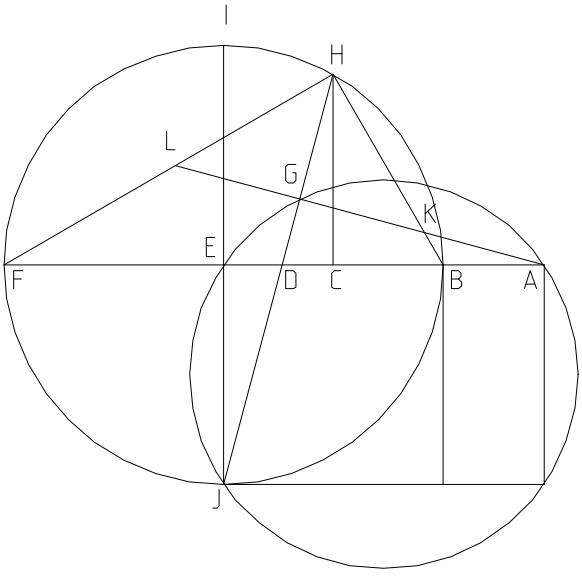
$JD := \sqrt{DE^2 + EJ^2} \quad DG := \frac{DE \cdot AD}{JD}$

$AG := \sqrt{AD^2 - DG^2} \quad GH := JH - (JD + DG)$

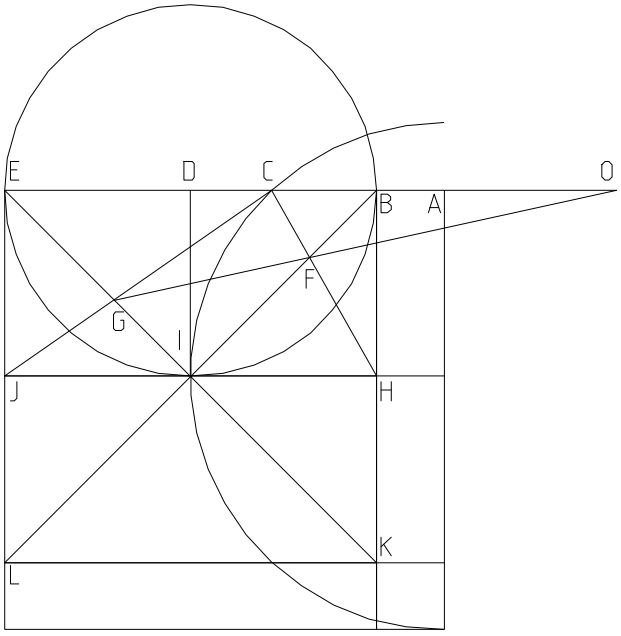
$HK := \sqrt{2 \cdot GH^2} \quad BH := \sqrt{BC^2 + CH^2} \quad HL := HK$

$BK := BH - HK \quad FH := \sqrt{CF^2 + CH^2}$

$FL := FH - HL$



$\frac{FL}{BK} = 4.333 \qquad \frac{AF}{AB} = 4.333$



Projecting from KL or HJ is productive, can I find any other productive points?

$AB := 5 \quad BE := 3 \quad BK := BE \quad AE := AB + BE$

$BD := \frac{BE}{2} \quad AC := \sqrt{AB \cdot AE} \quad HI := BD \quad IJ := BD$

$BC := AC - AB \quad CE := BE - BC$

$\Delta := 2 \quad \delta := 1 .. \Delta$

$BH_\delta := \frac{BK}{\Delta} \cdot \delta \quad Ha_\delta := \frac{BH_\delta \cdot HI}{BC} \quad EJ_\delta := BH_\delta$

$Ba_\delta := Ha_\delta + BD \quad Bb_\delta := \frac{(BH_\delta)^2}{Ba_\delta} \quad Jc_\delta := \frac{EJ_\delta \cdot IJ}{CE}$

$Ec_\delta := Jc_\delta + BD \quad Ed_\delta := \frac{(EJ_\delta)^2}{Ec_\delta} \quad Ef_\delta := Bb_\delta$

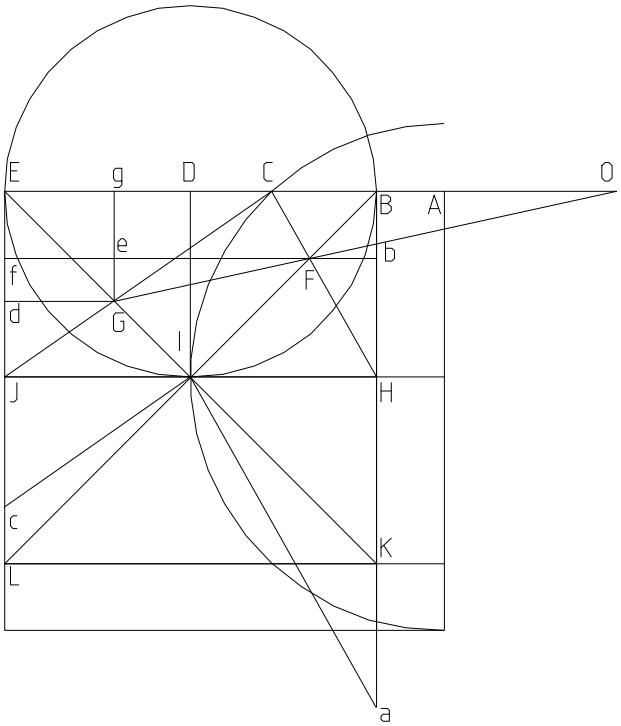
$df_\delta := Ed_\delta - Ef_\delta \quad Ge_\delta := df_\delta \quad Fb_\delta := \frac{HI \cdot BH_\delta}{Ba_\delta}$

$Gd_\delta := \frac{IJ \cdot EJ_\delta}{Ec_\delta} \quad ef_\delta := Gd_\delta \quad Fe_\delta := BE - (ef_\delta + Fb_\delta)$

$Gg_\delta := Ed_\delta \quad Og_\delta := \frac{Fe_\delta \cdot Gg_\delta}{Ge_\delta} \quad Eg_\delta := ef_\delta$

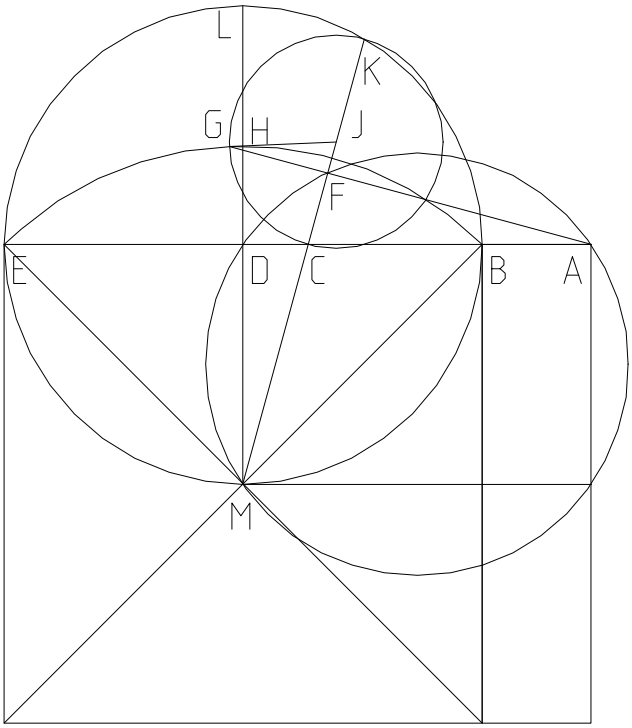
$EO_\delta := Og_\delta + Eg_\delta \quad BO_\delta := EO_\delta - BE$

$\left(\frac{EO_\delta}{BO_\delta}\right)$
1.2649
1.6



$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} = 1.2649 \qquad \frac{AE}{AB} = 1.6$

I have not found any.



The power line for cube root abstraction is developed off from a simple curve.

$AB := 33 \quad BE := 11 \quad BD := \frac{BE}{2} \quad AE := AB + BE$

$R_1 := \frac{\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AB}{2} \quad R_2 := \frac{AE - \left(AB \cdot AE^2\right)^{\frac{1}{3}}}{2}$

$D := \left[\left(AB \cdot AE^2\right)^{\frac{1}{3}} - \left(AB^2 \cdot AE\right)^{\frac{1}{3}}\right] + R_1 + R_2$

$BC := \frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D}$

The formula for the power line (BC) was given in 01_08_96.MCD

$CD := BD - BC \quad AC := BC + AB \quad DM := BD$

$DL := BD \quad CM := \sqrt{CD^2 + DM^2} \quad ML := DM + DL$

$MK := \frac{DM \cdot ML}{CM} \quad CK := MK - CM \quad CJ := \frac{CK}{2}$

$JG := CJ \quad JM := CM + CJ \quad BM := \sqrt{2 \cdot BD^2}$

$GM := BM \quad FJ := \frac{JG^2}{JM} \quad FM := JM - FJ$

$AD := BD + AB \quad HM := \frac{CM \cdot FM}{DM}$

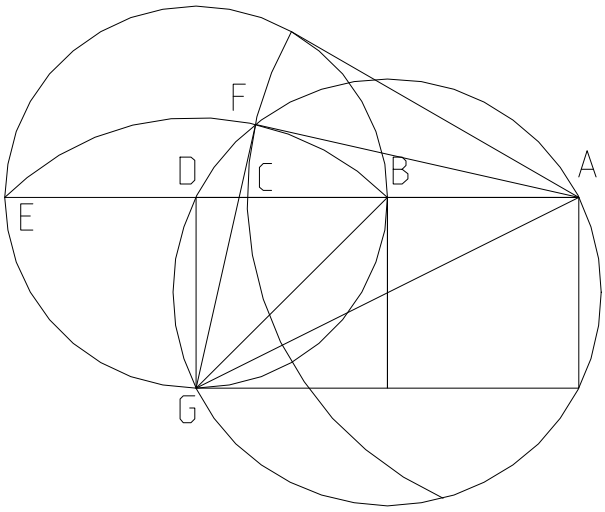
$DH := HM - DM \quad AH := \sqrt{DH^2 + AD^2}$

$CF := FM - CM \quad FH := \frac{CD \cdot HM}{CM}$

$AF_1 := AH - FH \quad AF_2 := \sqrt{AC^2 - CF^2}$

$AF_1 - AF_2 = 0$

Trivial Method; Square Root



$N := 9003$

$AE := N \quad AB := \frac{AE}{N} \quad BE := AE - AB \quad BD := \frac{BE}{2}$

$DG := BD \; AD := AB + BD \quad AG := \sqrt{AD^2 + DG^2}$

$BG := \sqrt{BD^2 + DG^2} \quad FG := BG \quad AF := \sqrt{AG^2 - FG^2}$

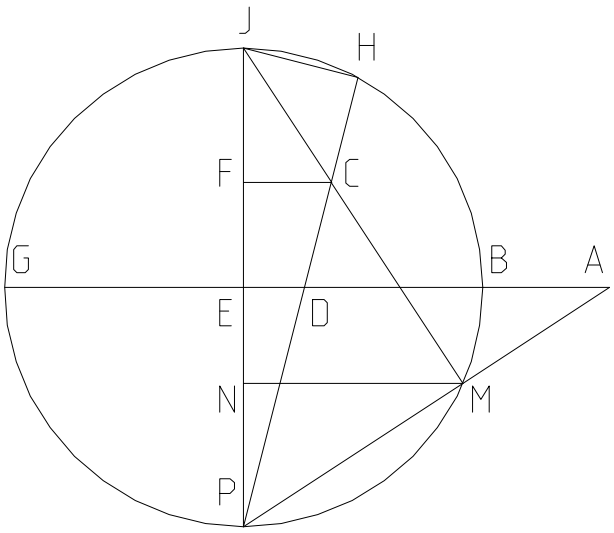
$AC := \sqrt{AB \cdot AE}$

$AC - AF = 0$

$AF = 94.884$

$AC = 94.884$

Given a point on BG, project to the point of cubic similarity.



$BG := 100 \quad BD := 49 \qquad BE := \frac{BG}{2}$

$EG := BE \quad DE := BE - BD \quad EP := BE$

$EJ := BE \quad DP := \sqrt{DE^2 + EP^2} \quad JP := EP + EJ$

$HP := \frac{EP \cdot JP}{DP} \quad DH := HP - DP \quad CD := \frac{DH}{2}$

$CP := DP + CD \quad CF := \frac{DE \cdot CP}{DP} \quad FP := \frac{EP \cdot CP}{DP}$

$EF := FP - EP \quad FJ := EJ - EF \quad CJ := \sqrt{CF^2 + FJ^2}$

$JM := \frac{FJ \cdot JP}{CJ} \quad JN := \frac{FJ \cdot JM}{CJ} \quad NP := JP - JN$

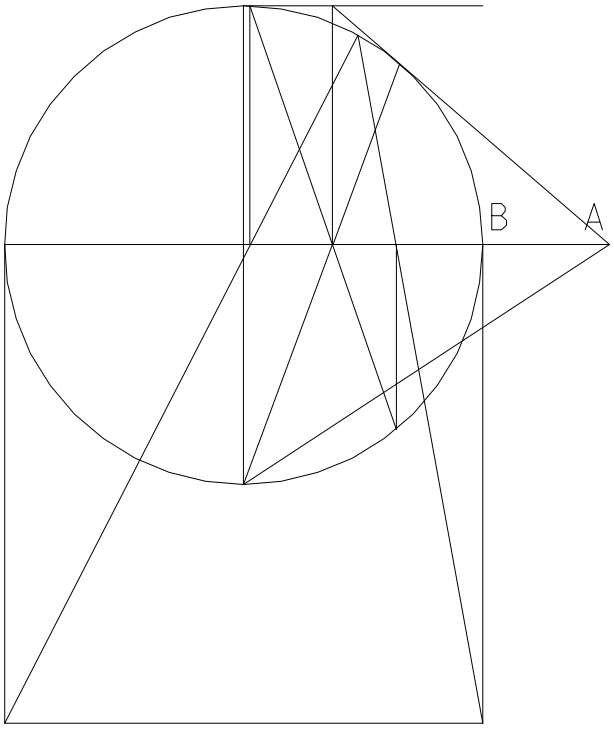
$MP := \frac{CF \cdot JP}{CJ} \quad AP := \frac{MP \cdot EP}{NP} \quad MN := \frac{CF \cdot JM}{CJ}$

$$\frac{BG^4 - 3 \cdot BG^3 \cdot BD + 3 \cdot BG^2 \cdot BD^2 - BD^3 \cdot BG}{BG^3 - 3 \cdot BG^2 \cdot BD + 3 \cdot BG \cdot BD^2 - 2 \cdot BD^3} = 884.222$$

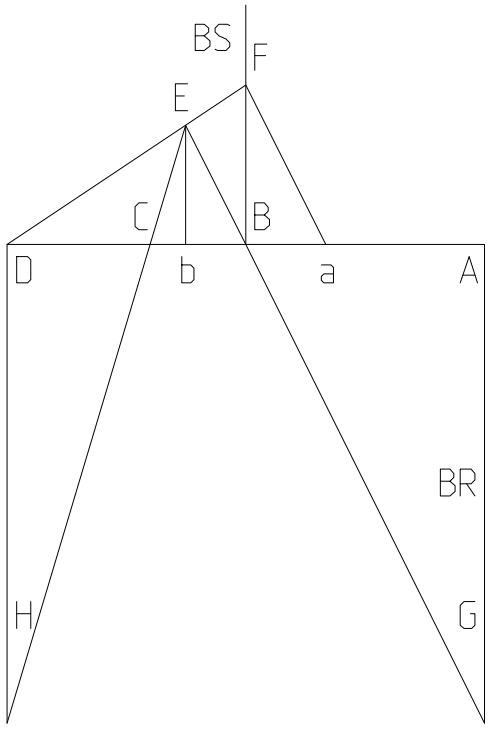
$AE := \frac{MN \cdot AP}{MP} \quad AG := AE + EG$

$$AG = 884.222$$

$$\frac{BG \cdot \left[(AG - BG)^{\frac{2}{3}} \cdot AG^{\frac{2}{3}} + AG^{\frac{1}{3}} \cdot BG - AG^{\frac{4}{3}} - BG \cdot (AG - BG)^{\frac{1}{3}} + (AG - BG)^{\frac{1}{3}} \cdot AG \right]}{(AG - BG)^{\frac{1}{3}} \cdot (2 \cdot AG - BG)} - BD = 0$$



One may be tempted to state the obvious, $\frac{A^{N+1}}{A^{N}} := A_t$, but what is not so obvious at first blush is that the processes themselves are assigned dimensional values. This has significance when using mathematics to theorize dimensions beyond three. Dimensions are so generally defined that processes are legitimate dimensional differences, but it is also impossible to defend mathematical theory about dimensions as objective. It becomes a point of Philosophical Mystic contemplation to realize that relationships concerning a single dimensional object and several processes adding dimensionally to the whole, is true of a multidimensional object without those processes!



Linear division $\frac{2\cdot(A + B)}{A}$

$BR := \frac{1}{4}$ $BS := 3$

$AD := \frac{BR}{BR}$ $AG := AD\cdot BR$ $AB := \frac{AD}{2}$

$BF := AB\cdot BS$ $Ba := \frac{AB\cdot BF}{AG}$ $BD := AB$

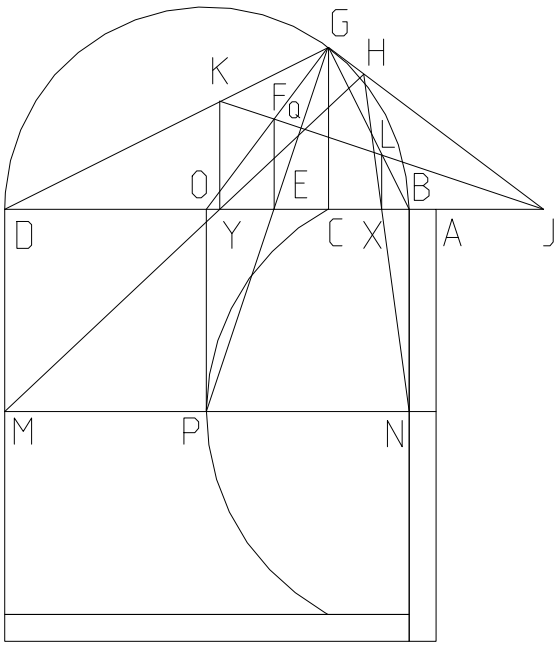
$Da := BD + Ba$ $Bb := \frac{Ba\cdot BD}{Da}$ $Db := BD - Bb$

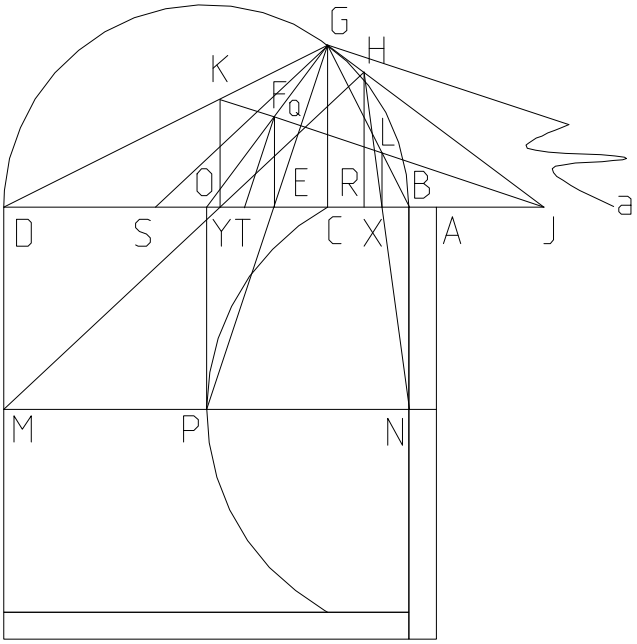
$Eb := \frac{BF\cdot Db}{BD}$ $DH := AG$

$DC := \frac{Db\cdot DH}{DH + Eb}$ $\frac{AD}{DC} = 26$

$\frac{2\cdot(BR + BS)}{BR} = 26$

Hitting JG from any BN while maintaining complimentary roots.





$AB := 2 \quad BD := 5 \quad BO := \frac{BD}{2} \quad AD := AB + BD$

$DO := BO \quad AC := \sqrt{AB \cdot AD} \quad BC := AC - AB$

$CD := BD - BC \quad CG := \sqrt{BC \cdot CD} \quad CO := BO - BC$

$N := 7 \quad OP := BO \cdot N \quad BN := OP \quad DM := OP$

$NP := BO \quad MP := BO \quad EO := \frac{CO \cdot OP}{OP + CG}$

$CE := CO - EO \quad EF := \frac{OP \cdot CE}{CO} \quad GO := BO$

$CJ := \frac{CG^2}{CO} \quad EJ := CJ + CE \quad Ca := \frac{EJ \cdot CG}{EF}$

$DJ := CD + CJ \quad Da := CD + Ca \quad JY := \frac{Ca \cdot DJ}{Da}$

$KY := \frac{EF \cdot JY}{EJ} \quad JK := \sqrt{JY^2 + KY^2} \quad JG := \sqrt{CJ^2 + CG^2}$

$GP := \sqrt{CO^2 + (OP + CG)^2} \quad EP := \sqrt{EO^2 + OP^2}$

$ET := \frac{EO \cdot EF}{OP} \quad JT := EJ + ET \quad FT := \sqrt{ET^2 + EF^2}$

$EG := GP - EP \quad EQ := \frac{FT \cdot EJ}{JT} \quad GQ := EG - EQ$

$KL := 2 \cdot GQ \quad JL := JK - KL$

$DY := DJ - JY \quad CS := \frac{DY \cdot CG}{DM} \quad JS := CS + CJ$

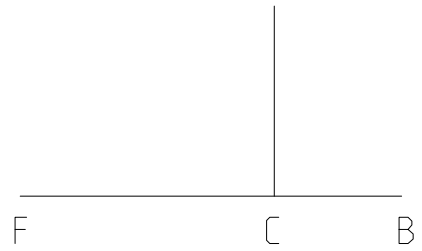
$YR := \frac{CS \cdot JY}{JS} \quad JR := JY - YR \quad HR := \frac{CG \cdot JR}{CJ}$

$AJ := CJ - AC \quad JX := \frac{JY \cdot JL}{JK}$

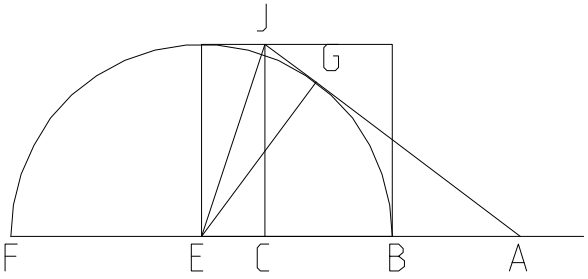
$BR_1 := JR - (AJ + AB) \quad BX := JX - (AJ + AB)$

$BR_2 := \frac{BX \cdot (BN + HR)}{BN}$

$BR_2 - BR_1 = 0$



Given BF and BC such that $\sqrt{(AB + BF)\cdot AB} = AB + BC$, find AB. It is obvious from the construction that answers are obtainable when BC is less than 1/2 of BF.

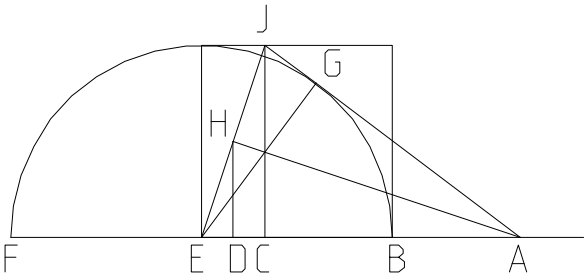


$\Delta := 5 \qquad \delta := 1..\Delta - 1 \quad BE := 5 \qquad BF := BE\cdot 2$

$BC_\delta := \frac{BE}{\Delta}\cdot\delta \quad CE_\delta := BE - BC_\delta \quad EG := BE$

$CJ := BE \quad EJ_\delta := \sqrt{\left(CE_\delta\right)^2 + CJ^2} \quad DH := \frac{CJ}{2}$

$DE_\delta := \frac{CE_\delta}{2} \quad AD_\delta := \frac{DH^2}{DE_\delta} \quad AE_\delta := AD_\delta + DE_\delta$



$AC_\delta := AE_\delta - CE_\delta \quad AJ_\delta := \sqrt{\left(AC_\delta\right)^2 + CJ^2}$

$GJ_\delta := \sqrt{\left(EJ_\delta\right)^2 - EG^2} \quad AB_\delta := AE_\delta - BE$

$AF_\delta := AB_\delta + BF$

$\left[\frac{1}{2}\cdot\frac{BE}{\Delta}\cdot\delta\cdot\frac{(-2\cdot\Delta + \delta)}{(-\Delta + \delta)}\right] - AC_\delta$

0
0
0
0

$GJ_\delta - CE_\delta$

0
0
0
0

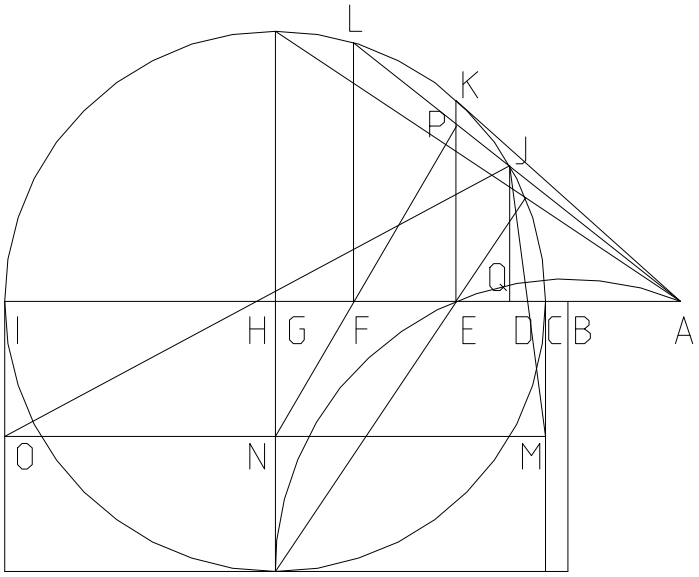
$AB_\delta + BC_\delta - \sqrt{AB_\delta\cdot(AB + BF)_\delta}$

0
0
0
0

$A := \frac{BF}{2} \qquad B := BC_2 \qquad \frac{2\cdot A\cdot B - B^2}{2\cdot A - 2\cdot B} - \left(AB_2 + BC_2\right) = 0$

$\sqrt{\left(AB_2 + BF\right)\cdot AB_2} - \left(AB_2 + BC_2\right) = 0$

$\frac{BF\cdot BC_2 - \left(BC_2\right)^2}{BF - 2\cdot BC_2} - \left(AB_2 + BC_2\right) = 0$ A rectangle minus a square divided by a line yeilds a line.



Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

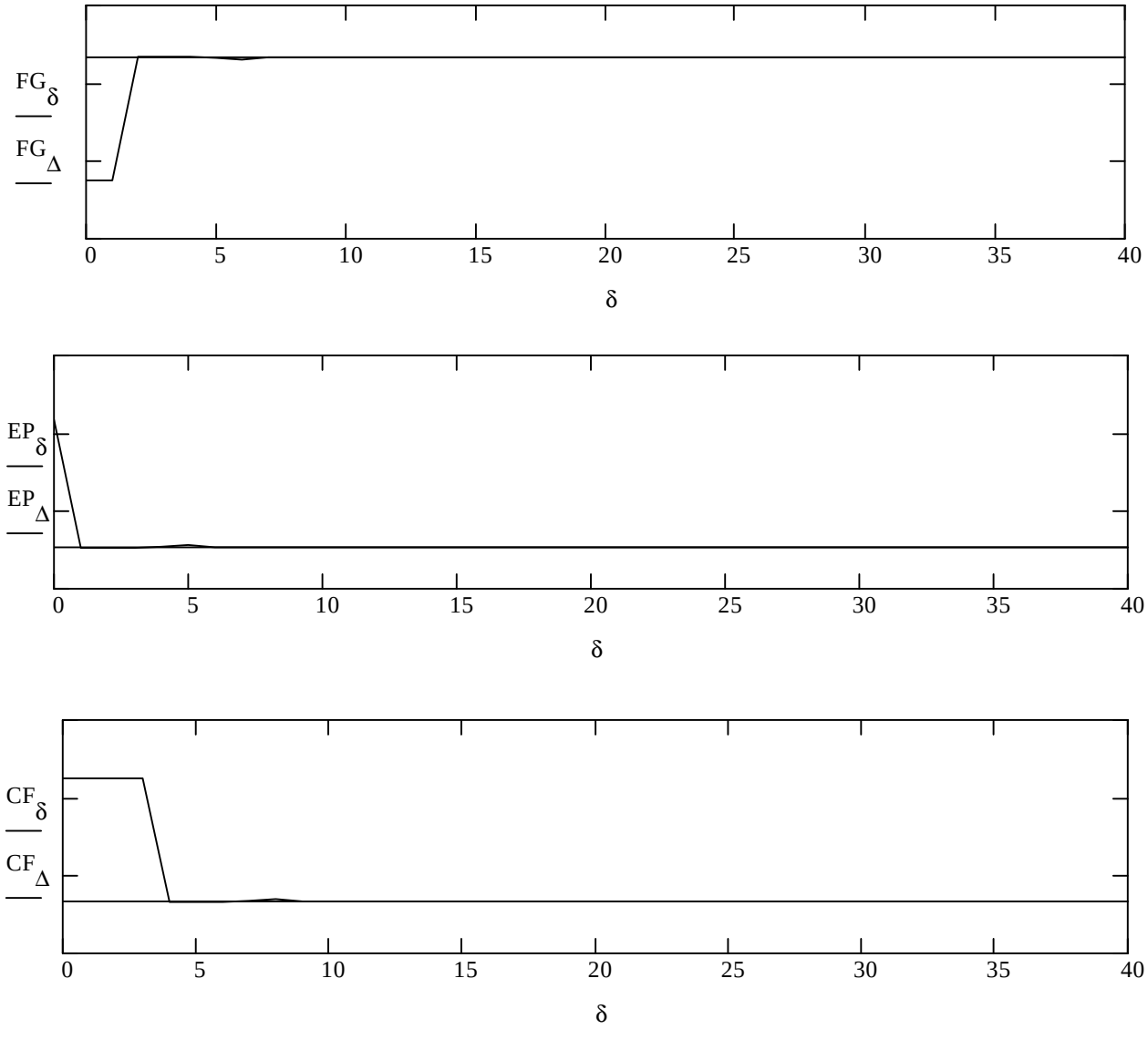
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

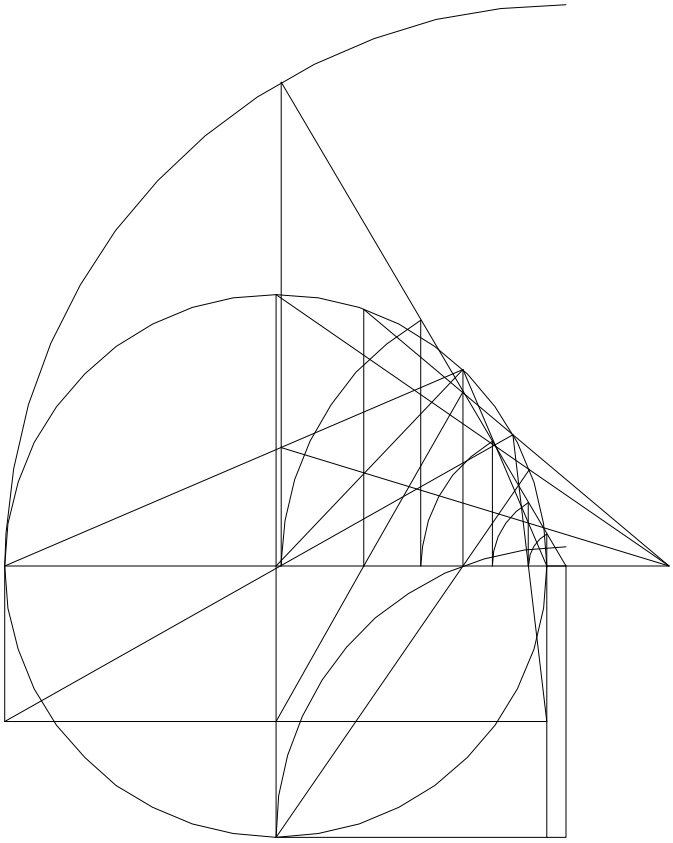
$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000$ $\left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$

$BD = 1.259921049894873$ $2^{\frac{1}{3}} = 1.259921049894873$

$BH = 1.587401051968199$ $4^{\frac{1}{3}} = 1.587401051968199$



The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.



Because of it's long projection, the last vertices is not drawn.

Does every n^{th} root series have at least one square root pair?

$n := 5 \qquad \delta := 0..\frac{n}{2}$

$A := 3 \quad B := 10$

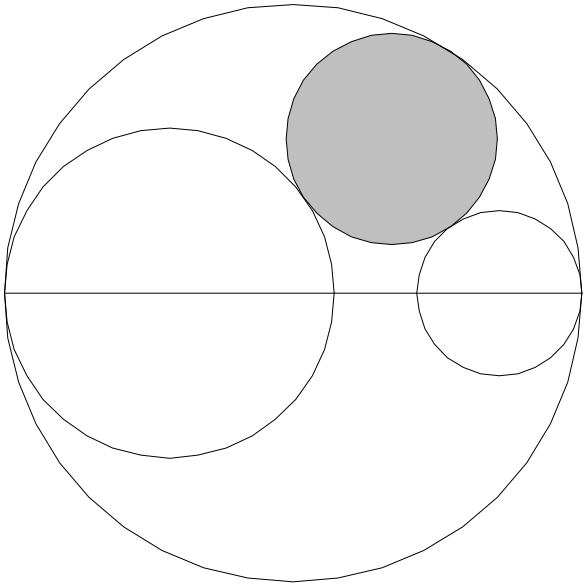
$$\sqrt{\left(A^{n-1}\cdot B^{1}\right)^{\frac{1}{n}}\cdot\left(A^{1}\cdot B^{n-1}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B} = 0$$

$$\sqrt{\left(A^{n-\delta}\cdot B^{\delta}\right)^{\frac{1}{n}}\cdot\left(A^{\delta}\cdot B^{n-\delta}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B}$$

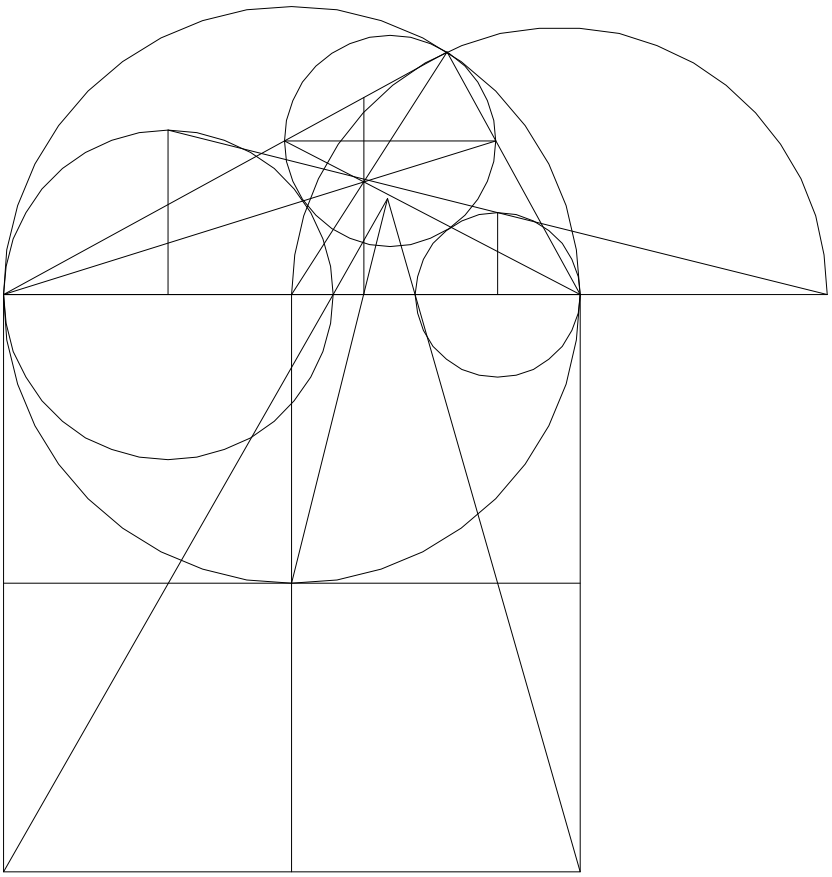
0
0
0

A root series has as many vertices on a circle as it has square root pairs, and it has the greater whole of n/2 vertices where n is the root series denominator.

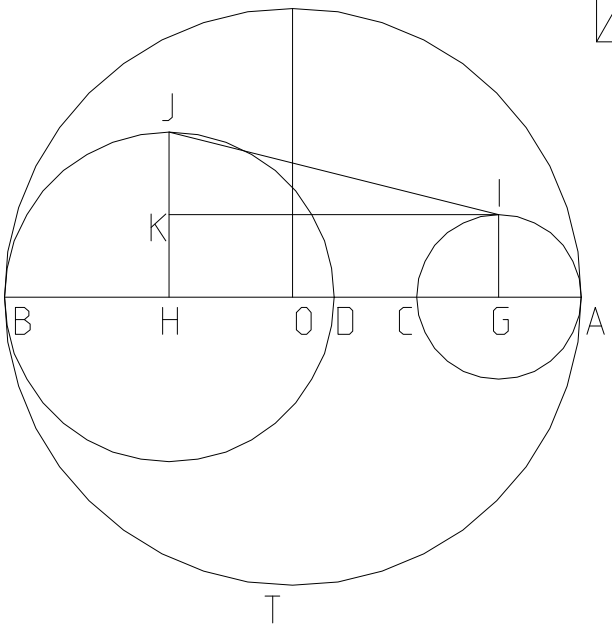
Method for Unequals



Given three circles in the said configuration, find the fourth.
I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."



Process Summary.

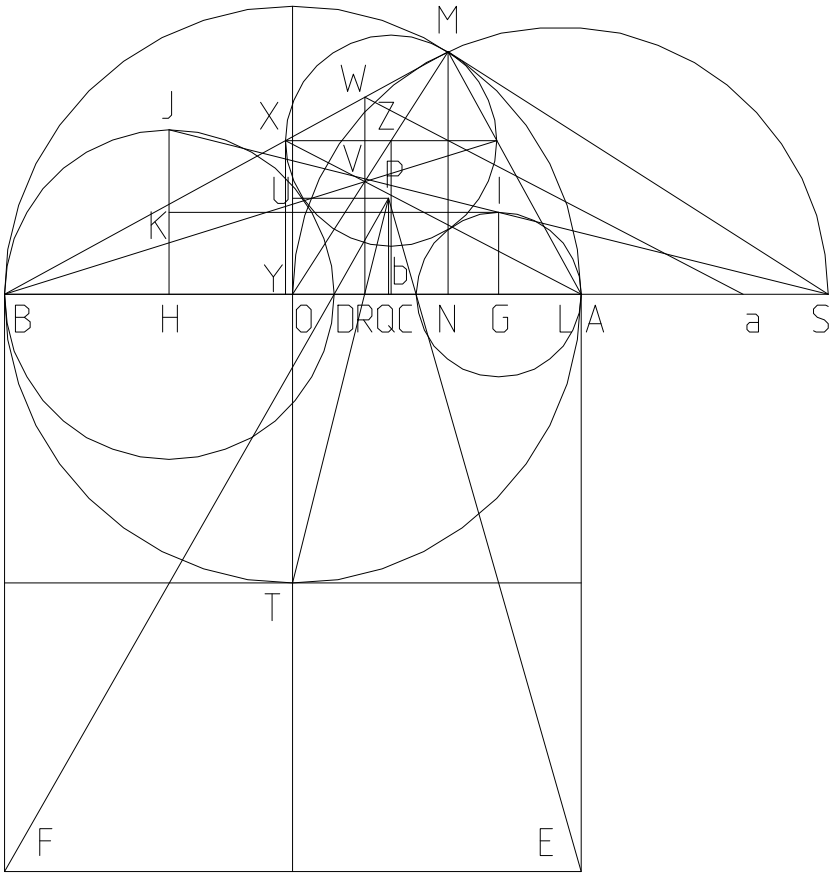


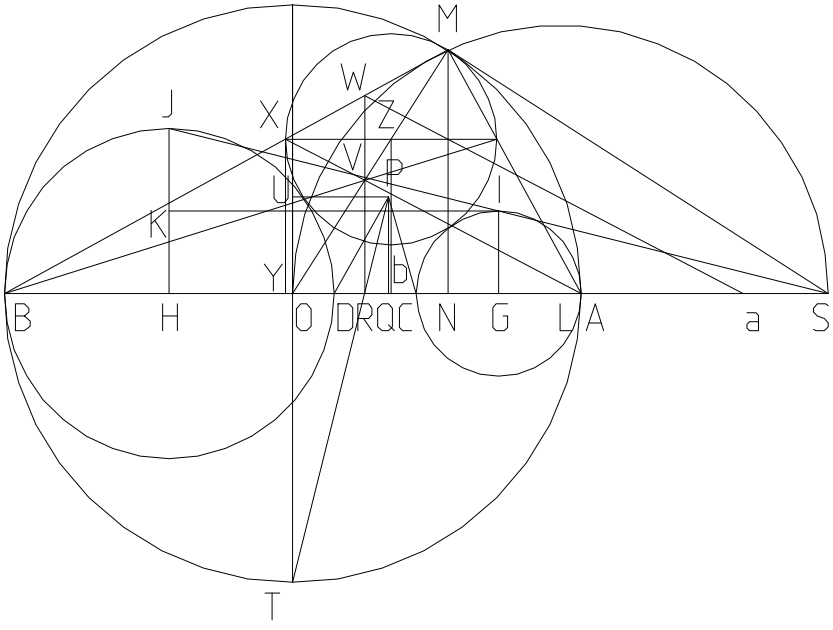
AO := 5 AG := 1 BH := 3 AB := 2·AO

BO := AO CG := AG GI := AG HJ := BH

DH := BH GH := AB – (AG + BH) HK := GI

JK := HJ – HK





$HS := \frac{GH \cdot HJ}{JK}$ $AH := AB - BH$

$AS := HS - AH$ $OS := AO + AS$

$SL := \frac{OS}{2}$ $MO := AO$ $MS := \sqrt{OS^2 - MO^2}$

$MN := \frac{MO \cdot MS}{OS}$ $NS := \frac{MS \cdot MN}{MO}$

$AN := NS - AS$ $ON := AO - AN$

$BN := AB - AN$ $AM := \sqrt{MN^2 + AN^2}$

$AE := AB$ $BF := AB$ $AC := 2 \cdot AG$

$BD := 2 \cdot BH$ $CD := AB - (AC + BD)$

$PQ := \frac{AE \cdot CD}{(AC + BD)}$ $OU := PQ$

$CQ := \frac{AC \cdot PQ}{AE}$ $AQ := AC + CQ$

$OQ := AO - AQ$ $OT := AO$

$TU := OT + OU$ $OR := \frac{OQ \cdot OT}{TU}$

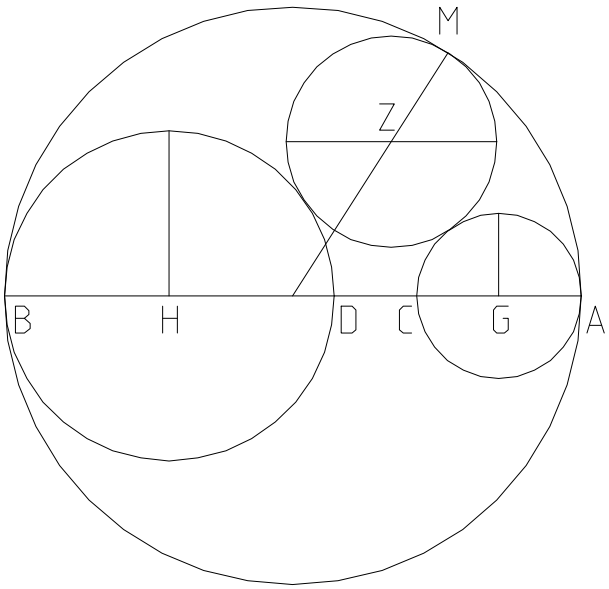
$RV := \frac{MN \cdot OR}{ON}$ $BR := BO + OR$

$RW := \frac{MN \cdot BR}{BN}$ $AR := AO - OR$

$Ra := \frac{AR \cdot RW}{RV}$ $XY := \frac{RW \cdot AB}{BR + Ra}$

$Zb := XY$ $OZ := \frac{MO \cdot Zb}{MN}$

$MZ := MO - OZ$ $MZ = 1.818$

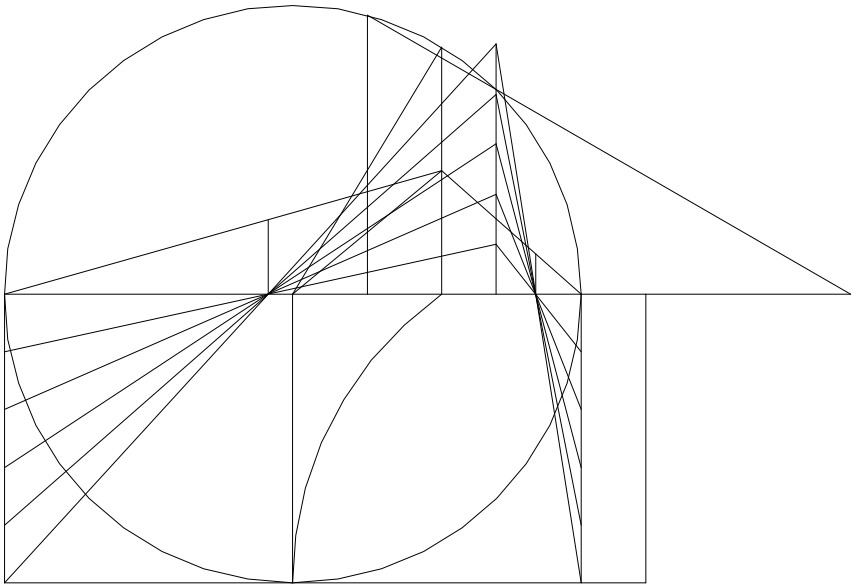


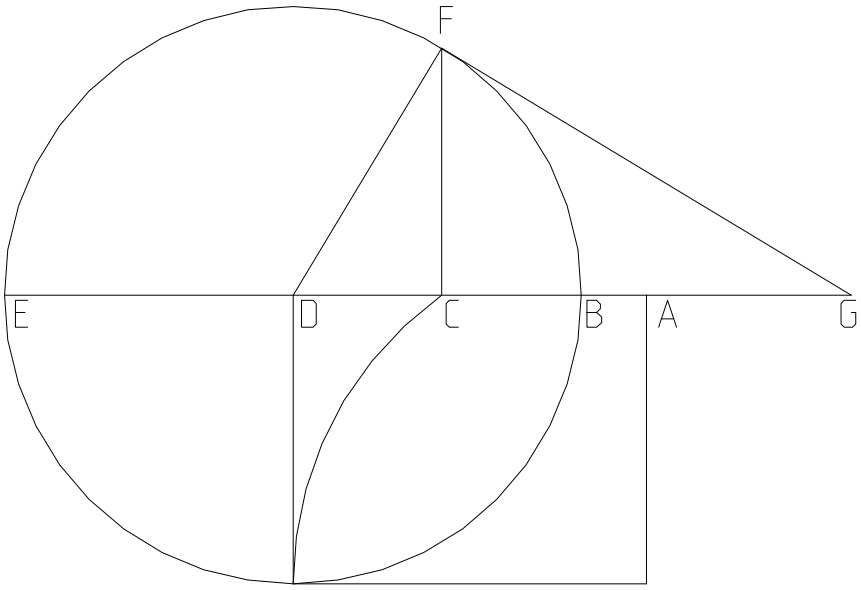
$$MZ = 1.818$$

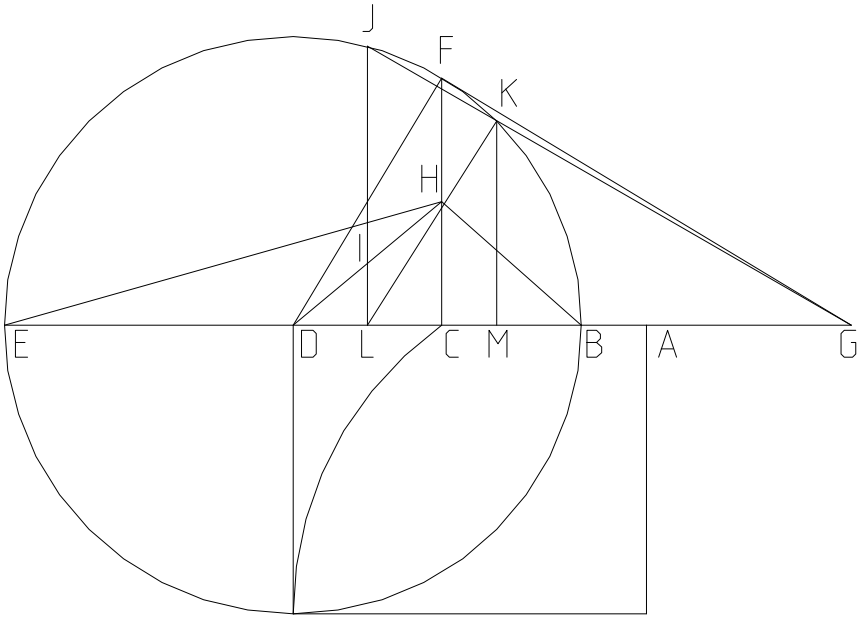
$$\frac{AB^3 - 2\cdot AB^2\cdot AG - 2\cdot AB^2\cdot BH + 4\cdot AB\cdot BH\cdot AG}{2\cdot AB^2 - 8\cdot BH\cdot AG} = 1.818$$

Neither in the process, nor in the Algebraic name. is the order of AG and BH recognized. Neither does it matter if they intersect.

On Gemini Roots







$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

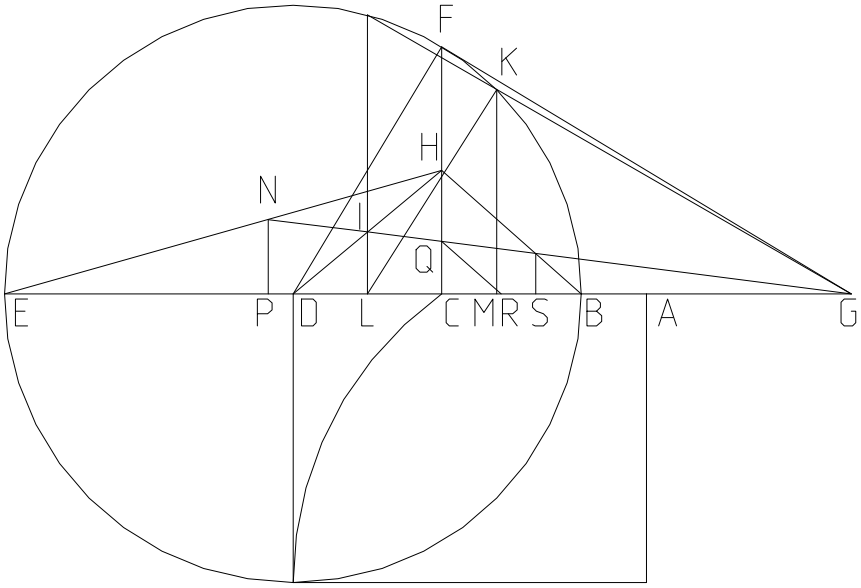
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

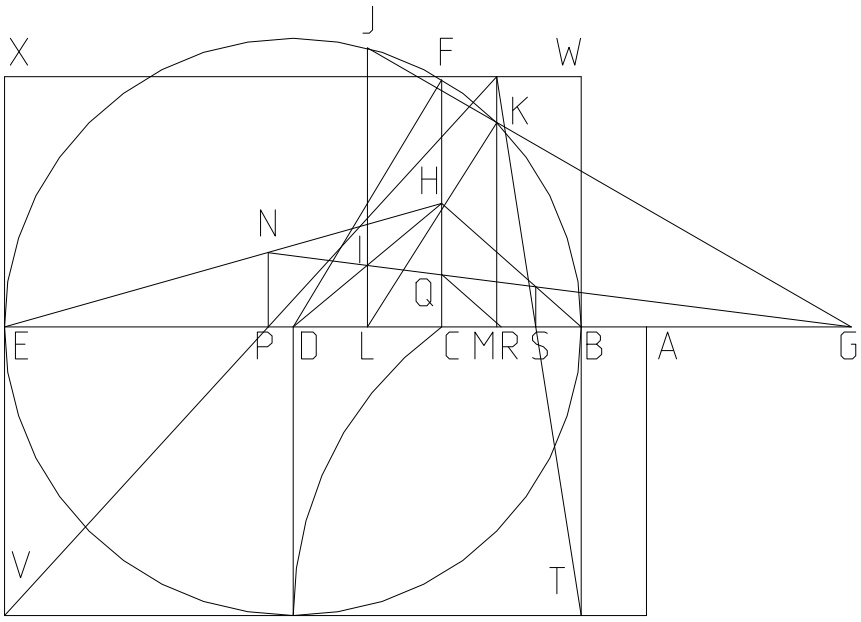


$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

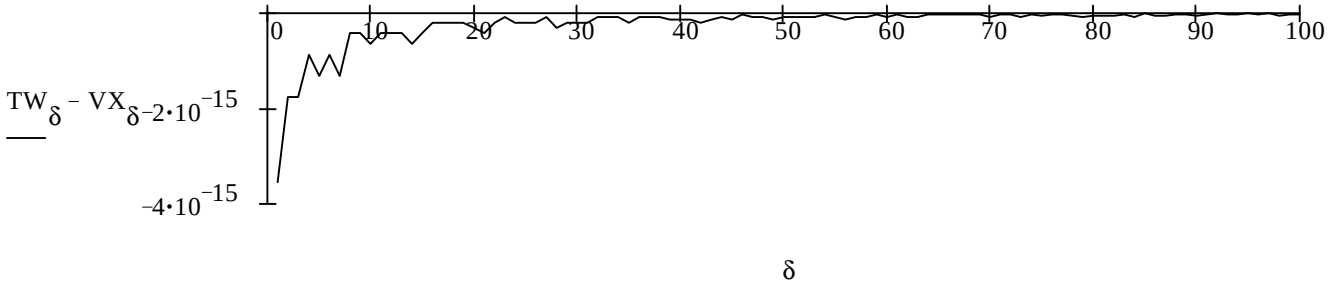
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$

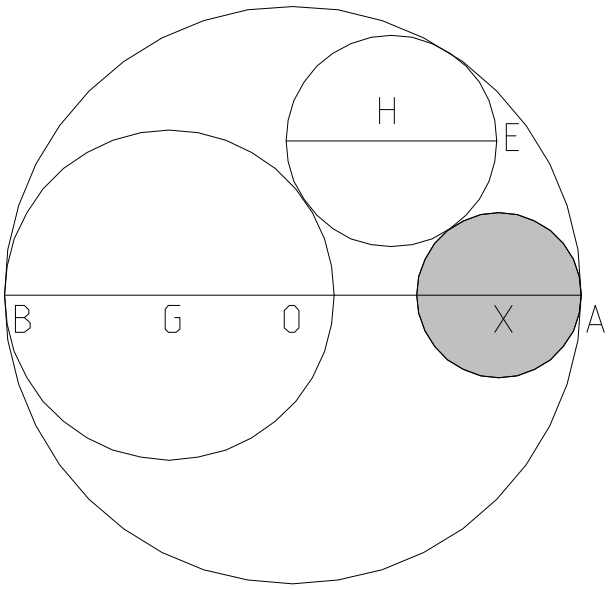


$$\delta := 1..100$$

$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$





04_16_96.MCD

Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO. This of course means that if the sum of BG and EH is equal to AO, we have no result.

$AO := 5 \quad BG := 2.5 \quad EH := 1.5$

$AB := 2 \cdot AO \quad BC := 2 \cdot BG \quad EF := 2 \cdot EH$

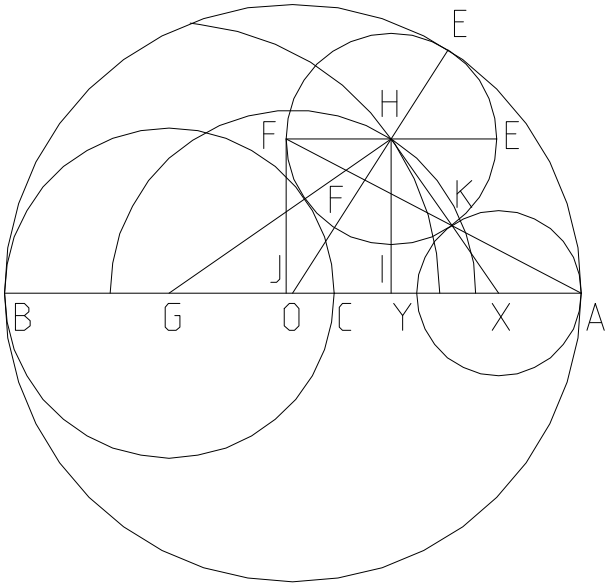
$GH := BG + EH \quad OH := AO - EH \quad GO := AO - BG$

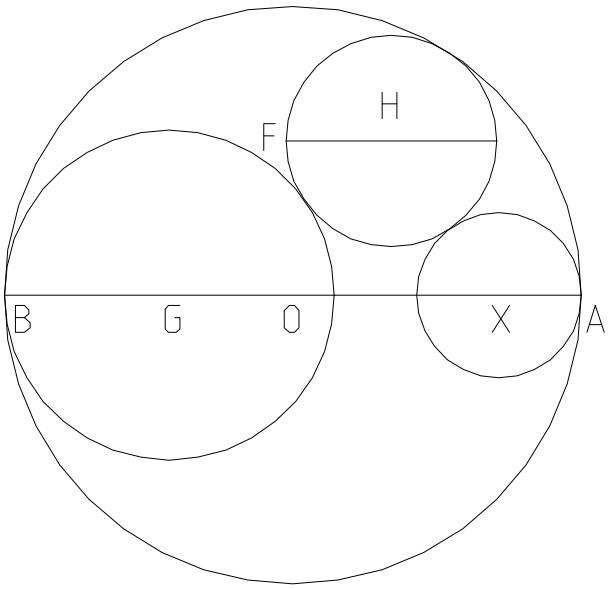
$GI := \frac{GH^2 + GO^2 - OH^2}{2 \cdot GO} \quad HI := \sqrt{GH^2 - GI^2}$

$AG := AB - BG \quad AI := AG - GI \quad IJ := EH$

$AJ := AI + IJ \quad FJ := HI \quad AF := \sqrt{AJ^2 + FJ^2}$

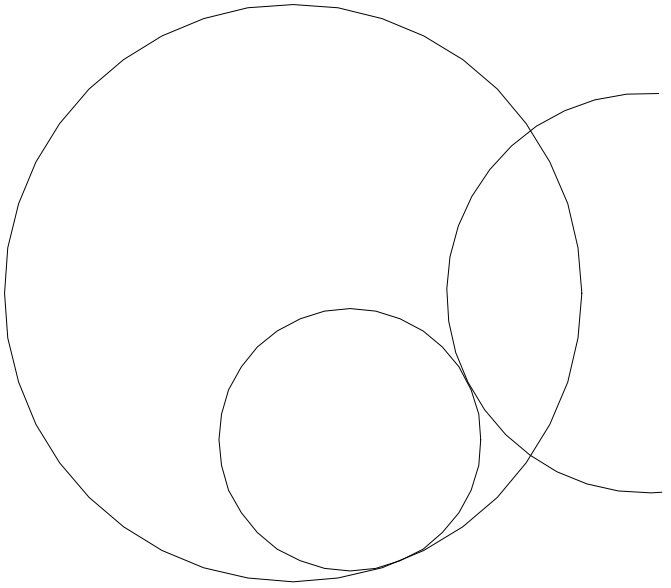
$FK := \frac{AJ \cdot EF}{AF} \quad AK := AF - FK \quad AY := \frac{AF \cdot AK}{AJ}$

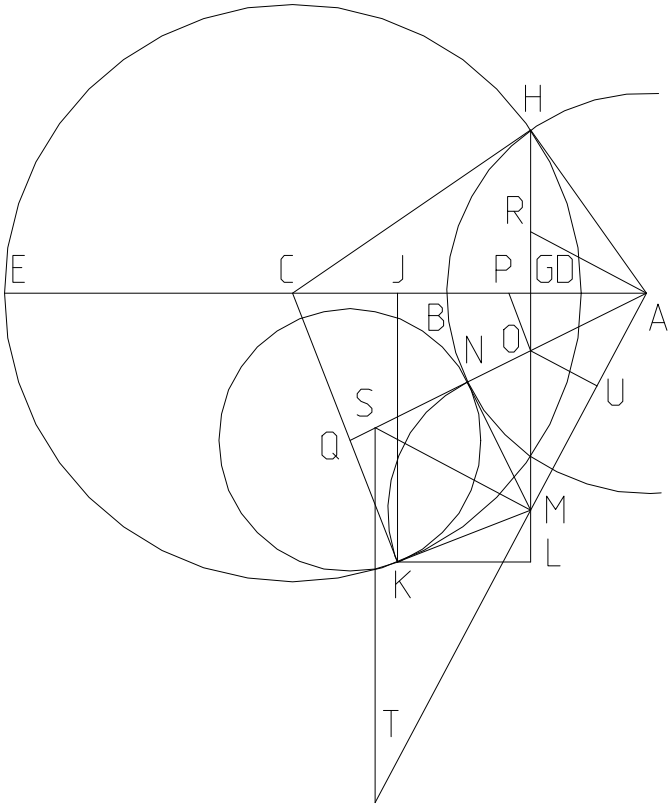




$AX := \frac{AY}{2} \quad AX = 2.857$

$\frac{AO^3 - AO^2 \cdot EH - AO^2 \cdot BG}{AO^2 - AO \cdot BG - EH \cdot BG} = 2.857$





Given a point of tangency, draw a circle in a crescent tangent to the other side. This figure is given for the tangent on the exterior of the crescent, the other will become obvious.

$AB := Concave_Radius$

$CD := Convex_Radius \quad AC := Center_Difference$

$DE := 2 \cdot CD \quad BF := 2 \cdot AB \quad CE := CD$

$AE := AC + CE \quad BE := AE - AB$

$$CG := \frac{CD^2 + AC^2 - AB^2}{2 \cdot AC} \quad AG := AC - CG$$

$GE := AE - AG$

$GJ := Power_Line_Tangent \cdot GE$

$EJ := GE - GJ$

$DJ := DE - EJ \quad JK := \sqrt{DJ \cdot EJ}$

$CJ := CG - GJ \quad CK := CD \quad KL := GJ$

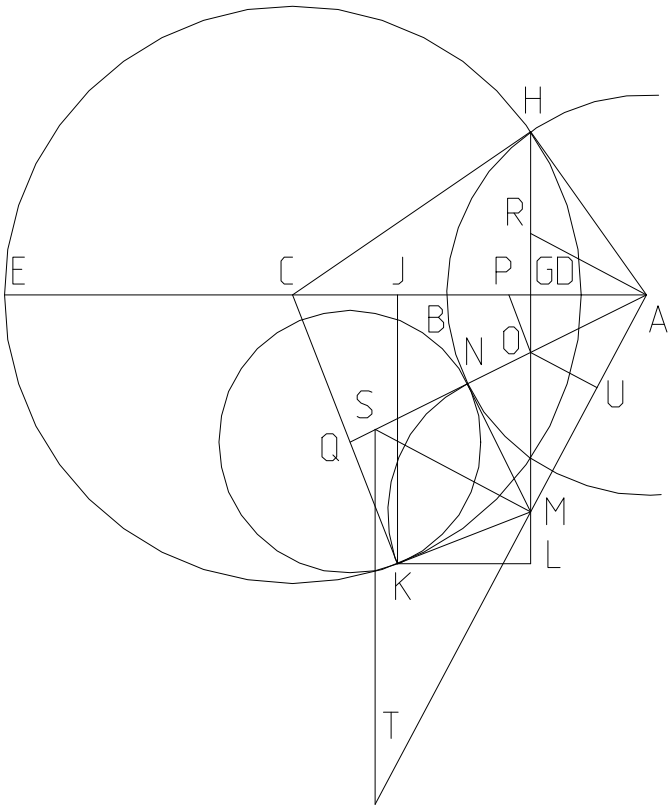
$$LM := \frac{CJ \cdot KL}{JK} \quad GL := JK \quad GM := GL - LM$$

$$AM := \sqrt{AG^2 + GM^2} \quad AN := AB$$

$$AS := \frac{AM^2}{AN} \quad MR := \frac{AM^2}{GM}$$

$$MS := \sqrt{AS^2 - AM^2} \quad AR := \sqrt{MR^2 - AM^2}$$

$$ST := \frac{MR \cdot MS}{AR} \quad MT := \frac{AM \cdot MS}{AR}$$



$$MO := \frac{ST \cdot AM}{AM + MT}$$

$$GO := GM - MO \quad GP := \frac{CJ \cdot GO}{JK}$$

$$AP := AG + GP \quad OP := \frac{CK \cdot GO}{JK}$$

$$CQ := \frac{OP \cdot AC}{AP} \quad QK := CK - CQ$$

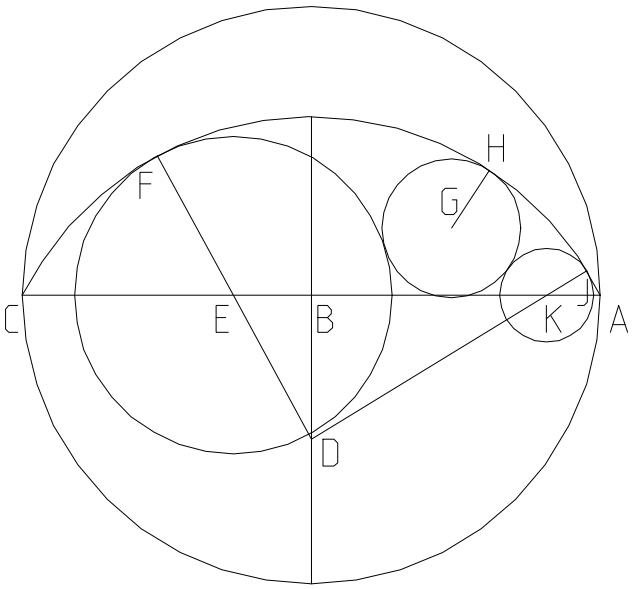
$QK = 0.206$

Concave_Radius $\equiv 2.37$

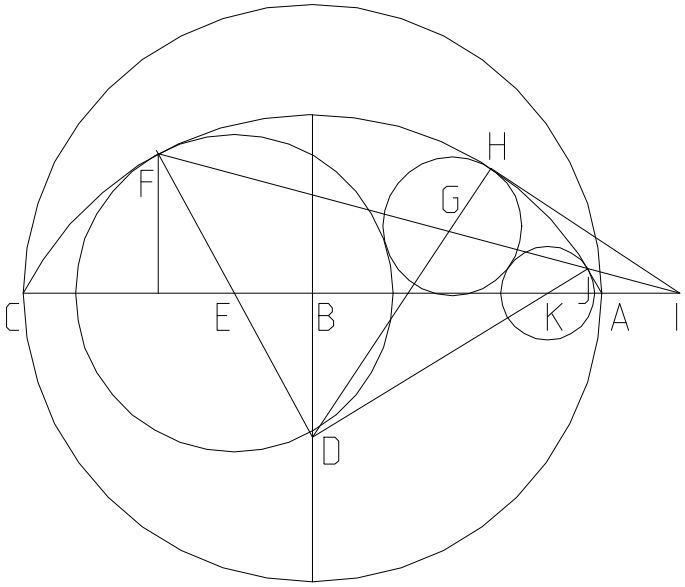
Convex_Radius $\equiv 1.5$

Center_Difference $\equiv 1.84$

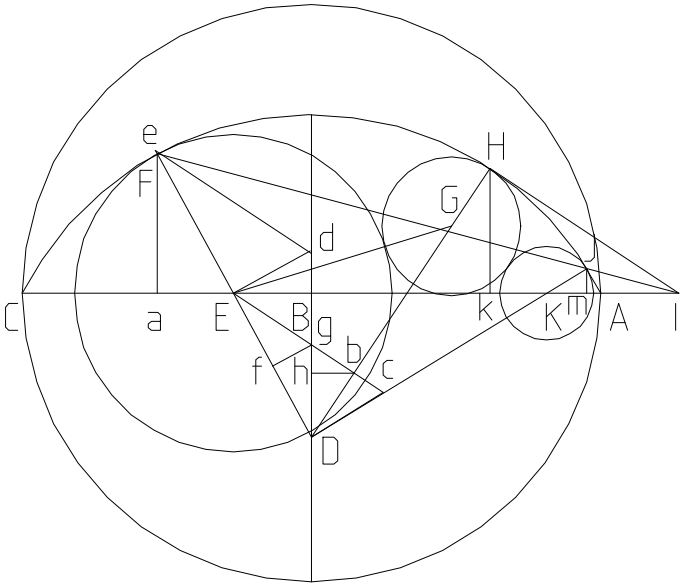
Power_Line_Tangent $\equiv \frac{1}{3}$ Given as Fraction < 1.



Place EF and GH and find JK.



Process summary



$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

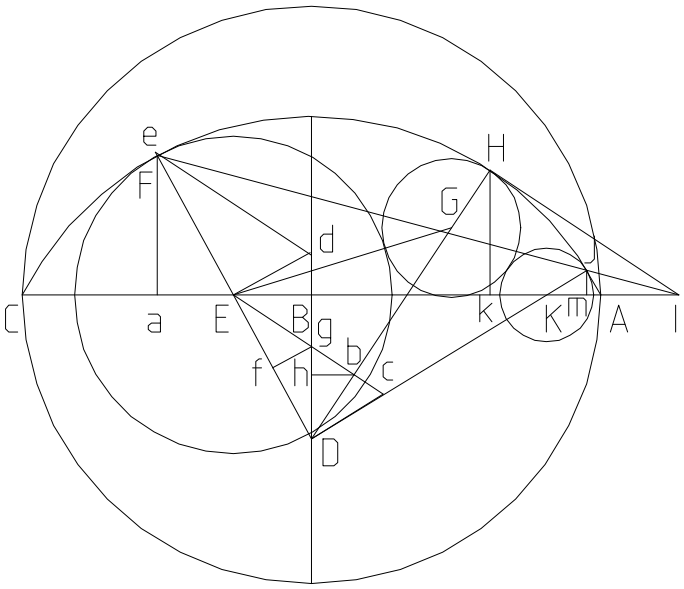
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$



$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

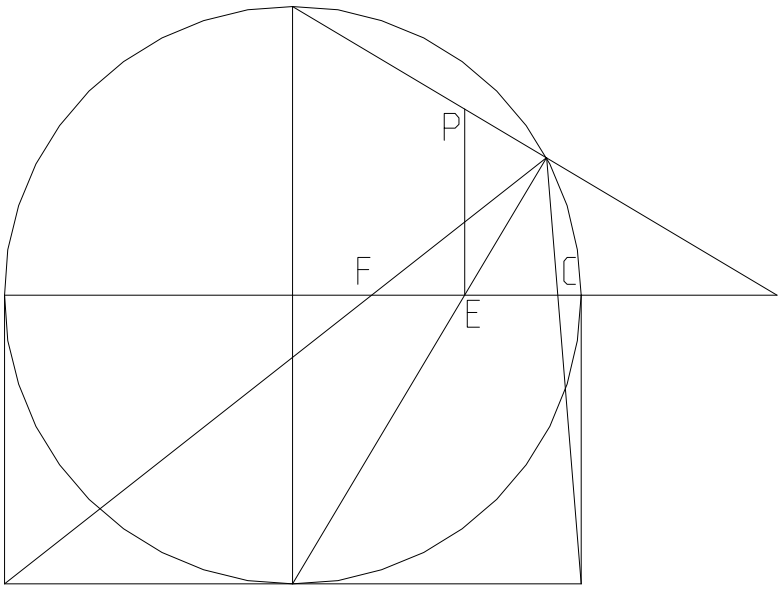
$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

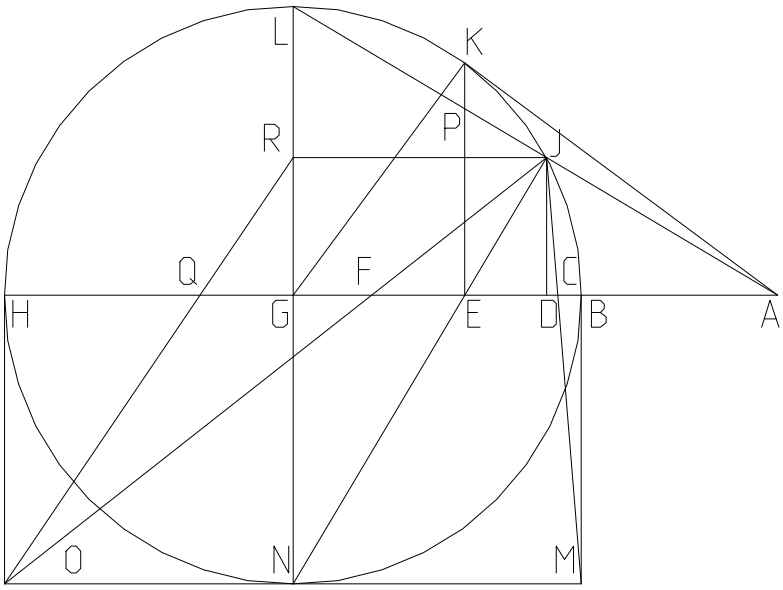
$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.



I found this little sketch in my notebook and have no idea of when I did it or why.

Does CF always equal EP?



$\Delta := 100 \quad \delta := 1..\Delta - 1 \quad BG := 10 \quad GH := BG$

$GN := BG \quad BD_\delta := \frac{BG}{\Delta}\cdot\delta \quad DG_\delta := BG - BD_\delta$

$DH_\delta := GH + DG_\delta \quad DJ_\delta := \sqrt{BD_\delta \cdot DH_\delta}$

$EG_\delta := \frac{DG_\delta \cdot GN}{GN + DJ_\delta} \quad BE_\delta := BG - EG_\delta$

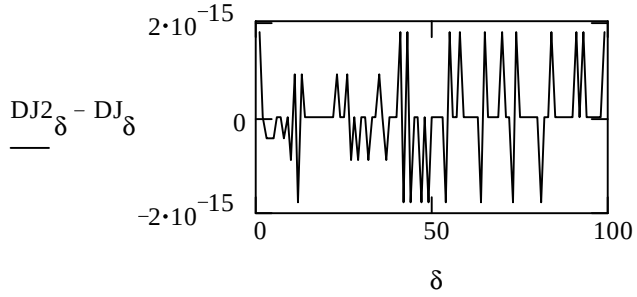
$EH_\delta := EG_\delta + GH \quad EK_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

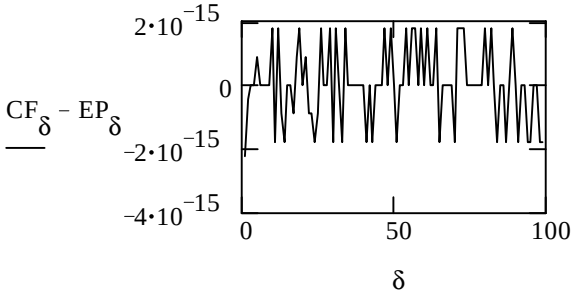
$AE_\delta := \frac{(EK_\delta)^2}{EG_\delta} \quad AB_\delta := AE_\delta - BE_\delta$

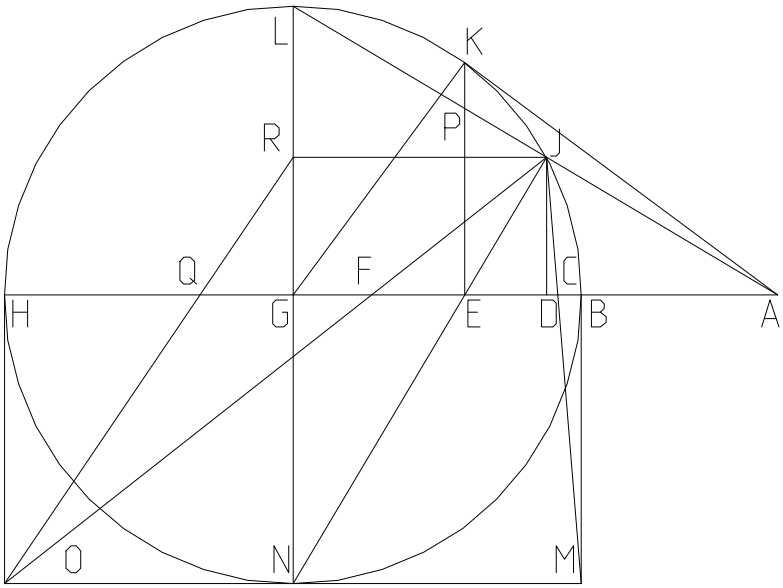
$GL := BG \qquad AD_\delta := AB_\delta + BD_\delta$

$AG_\delta := AB_\delta + BG \qquad DJ2_\delta := \frac{GL \cdot AD_\delta}{AG_\delta}$

$EP_\delta := \frac{GL \cdot AE_\delta}{AG_\delta} \quad GR_\delta := DJ_\delta \quad NO := BG \quad NR_\delta := GN + GR_\delta \quad GQ_\delta := \frac{NO \cdot GR_\delta}{NR_\delta} \quad CF_\delta := 2 \cdot GQ_\delta$







Reducing both by the symbolic processor leaves a little.

$$BG := 10 \qquad BD := \frac{BG}{13}$$

$$CF := 2\cdot BG\cdot\sqrt{BD}\cdot\frac{\sqrt{2\cdot BG - BD}}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)}$$

$$EP := -2\cdot BG\cdot\frac{\left(-BG\cdot\sqrt{BD}\cdot\sqrt{2\cdot BG - BD} - 2\cdot BG\cdot BD + BD^2\right)}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)^2}$$

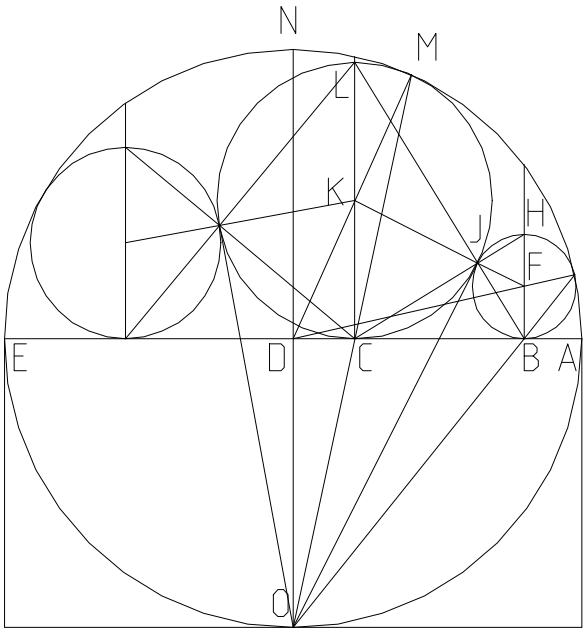
$EP = 5.556 \qquad EP - CF = 0$

And if I divide one by the other and reduce; $\dfrac{\left[BG\cdot\sqrt{2\cdot BG - BD} + 2\cdot BG\cdot\sqrt{BD} - BD^{\left(\frac{3}{2}\right)}\right]}{\left[\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)\cdot\sqrt{2\cdot BG - BD}\right]} = 1$

This is another figure that I had sketched out last year but never got around to writing down.

Given a Circle, place the next on the diameter.

I tried to reduce this series with the symbolic processor, but it is having trouble, at some point it switches AC for EC and I get the other circle.



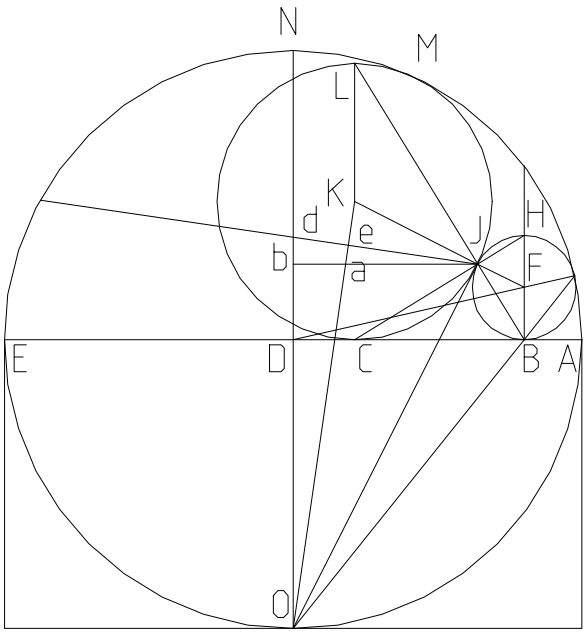
$AD := Radius \quad AE := 2 \cdot AD \quad AC := \frac{AE}{N}$

$CD := AD - AC \quad DO := AD \quad CO := \sqrt{DO^2 + CD^2}$

$NO := AE \quad MO := \frac{DO \cdot NO}{CO} \quad CM := MO - CO$

$CK := \frac{DO \cdot CM}{MO} \quad KO := \sqrt{CD^2 + (DO + CK)^2}$

$JK := CK$

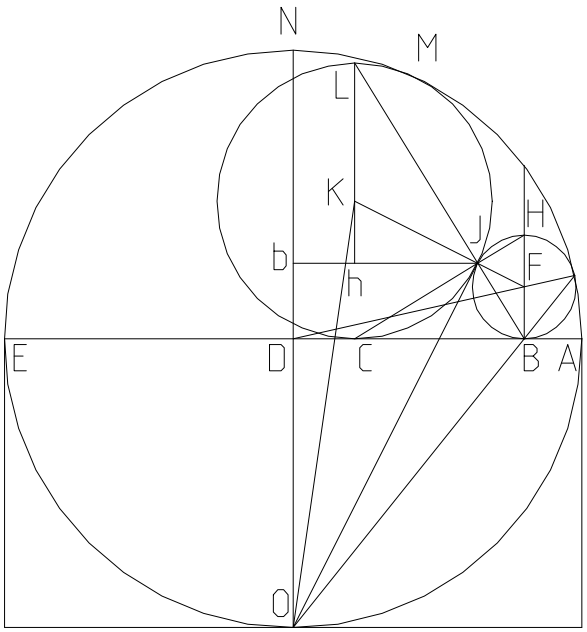


$Ke := \frac{JK^2}{KO} \quad Oe := KO - Ke \quad de := \frac{CD \cdot Oe}{DO + CK}$

$Je := \sqrt{JK^2 - Ke^2}$

$Jd := de + Je \quad bd := \frac{CD \cdot Jd}{KO}$

$Od := \frac{KO \cdot de}{CD} \quad Db := Od - DO - bd$



$Kh := CK - Db$

$Lh := CK + Kh \qquad FJ := \frac{JK \cdot Db}{Lh}$

$N \equiv \frac{10}{1} \qquad FJ = 3.965$

$Radius \equiv 108 \qquad CK = 19.44$

And from the other side;

$N \equiv \frac{10}{9} \qquad FJ = 51.53$

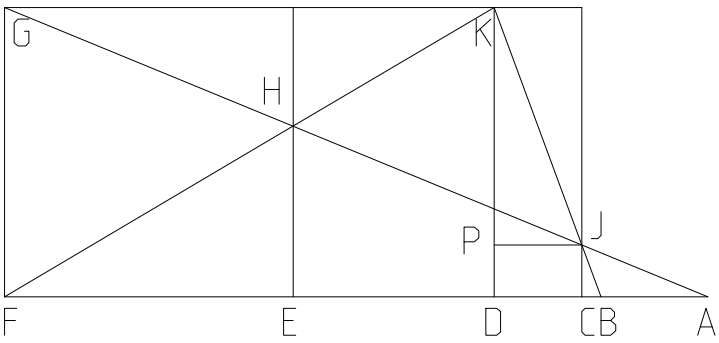
$Radius \equiv 108 \qquad CK = 19.44$

This figure might be recognized as the similarity point for Gemini root projection.

Given AC, CF, and that

$$AC = -CF \cdot \frac{\left(\sqrt{BC} \cdot \sqrt{BC + CF} - BC\right)}{\left(-CF + 2 \cdot \sqrt{BC} \cdot \sqrt{BC + CF} - 2 \cdot BC\right)}$$

find BC.



N can be any value whatever, except 0.

$CF := 216 \quad AC := 47.29 \quad N := 100000$

$CE := \frac{CF}{2} \quad AF := AC + CF \quad AE := AC + CE$

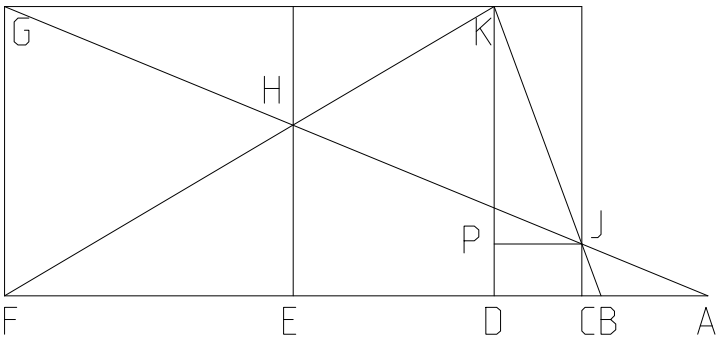
$FG := N \quad EH := \frac{FG \cdot AE}{AF} \quad EF := CE$

$DF := \frac{EF \cdot FG}{EH} \quad CJ := \frac{FG \cdot AC}{AF} \quad DP := CJ$

$CD := CF - DF \quad DK := FG \quad KP := DK - DP$

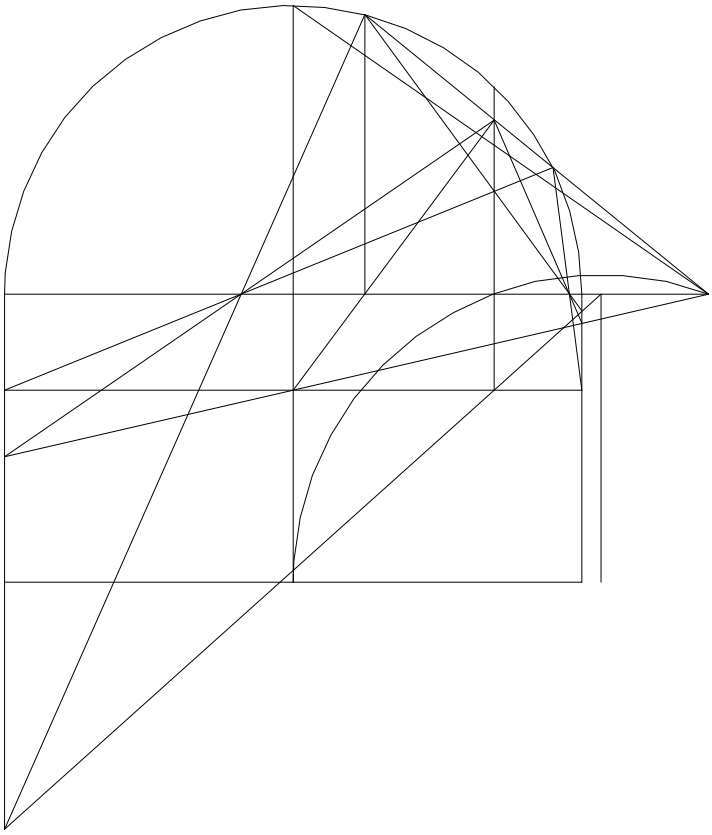
$BD := \frac{CD \cdot DK}{KP} \quad BD = 40.089$

$BC := BD - CD \quad BC = 7.201$



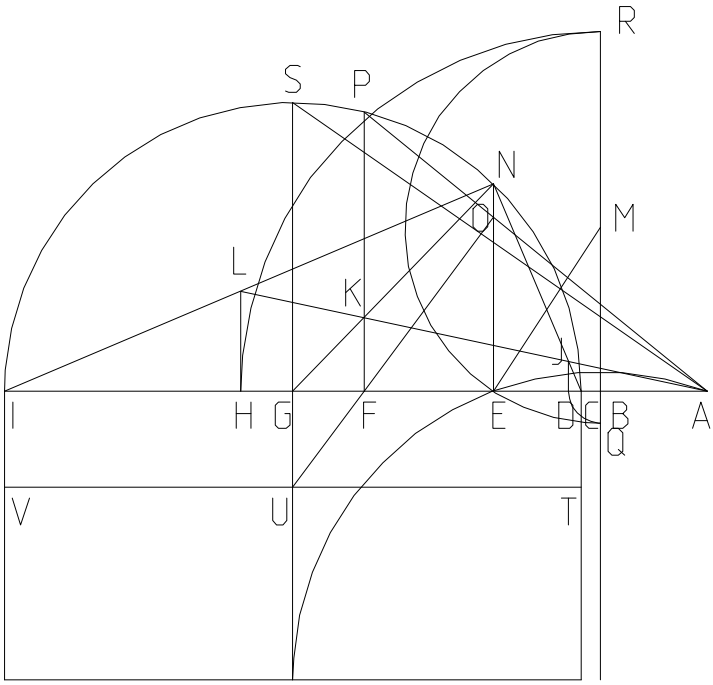
$$AC_f := -CF \cdot \frac{\left(\sqrt{BC} \cdot \sqrt{BC + CF} - BC\right)}{\left(-CF + 2 \cdot \sqrt{BC} \cdot \sqrt{BC + CF} - 2 \cdot BC\right)}$$

$$\frac{AC_f}{AC} = 1$$



Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.



$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

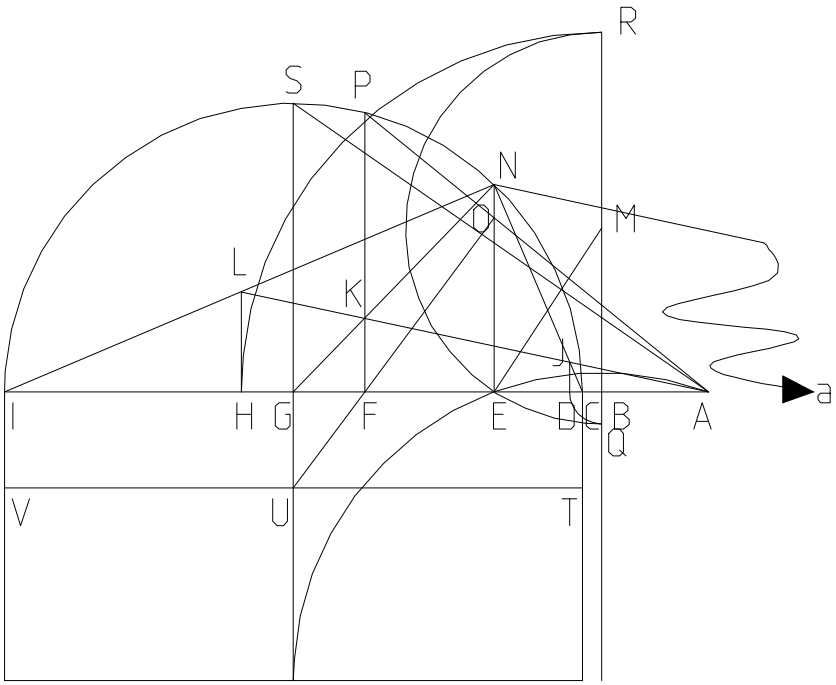
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



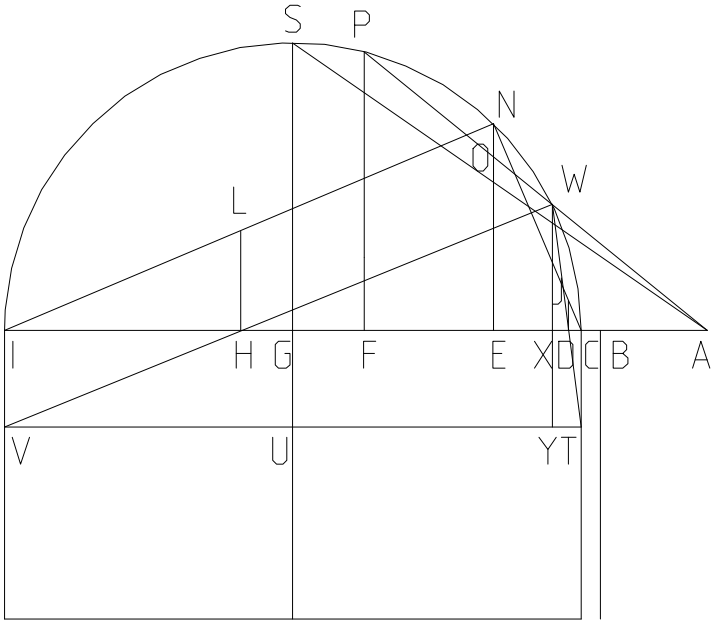
$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$



$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

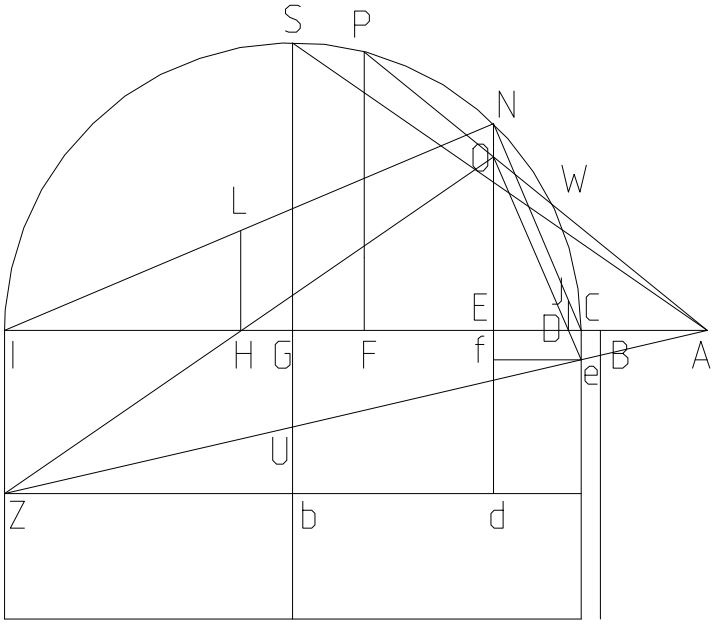
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

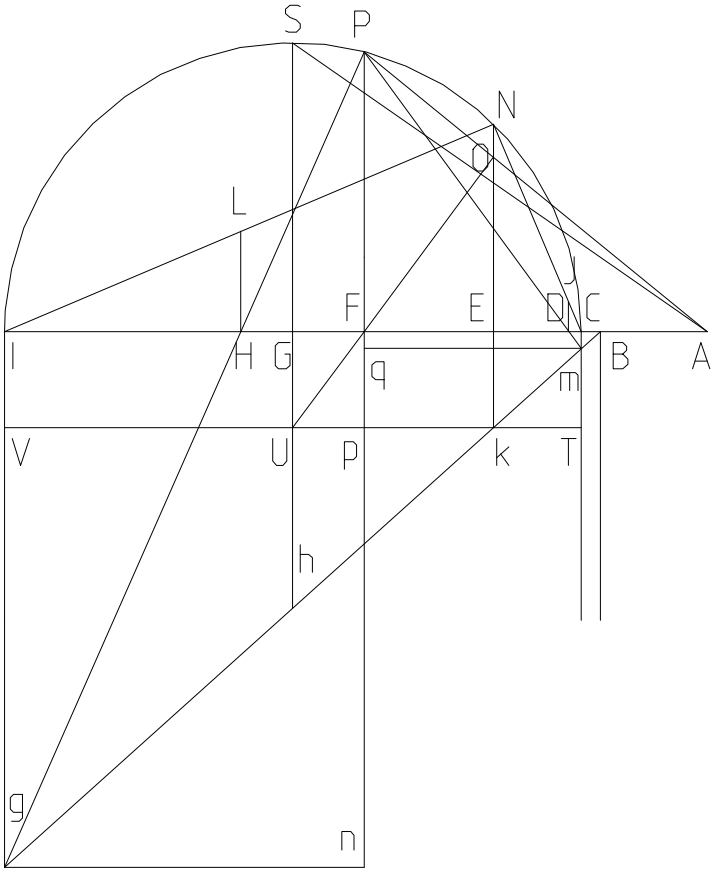
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$



$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



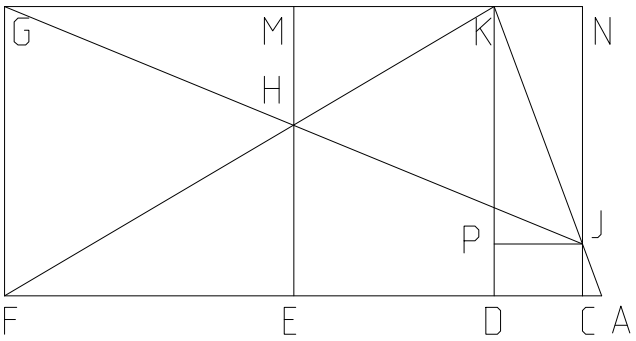
$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

Given CF and CD such that CD = $\sqrt{AF \cdot AC} - AC$ find AC.



$CF := 216 \quad CD := 32.89 \quad CE := \frac{CF}{2} \quad EF := CE$

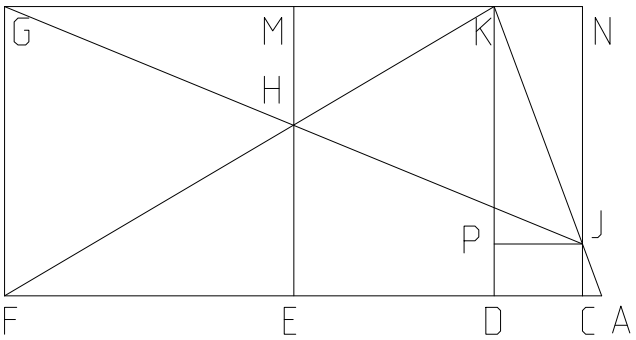
Except for 0, N can be any value.

$N := 108 \quad FG := N \quad DK := N \quad DF := CF - CD$

$EH := \frac{DK \cdot EF}{DF} \quad EM := FG \quad HM := EM - EH$

$CN := FG \quad HM := EM - EH \quad GN := CF$

$GM := EF \quad JN := \frac{HM \cdot GN}{GM} \quad JC := CN - JN$



$KP := JN \quad JP := CD \quad AD := \frac{JP \cdot DK}{KP}$

$AC := AD - CD \quad AC = 7.201$

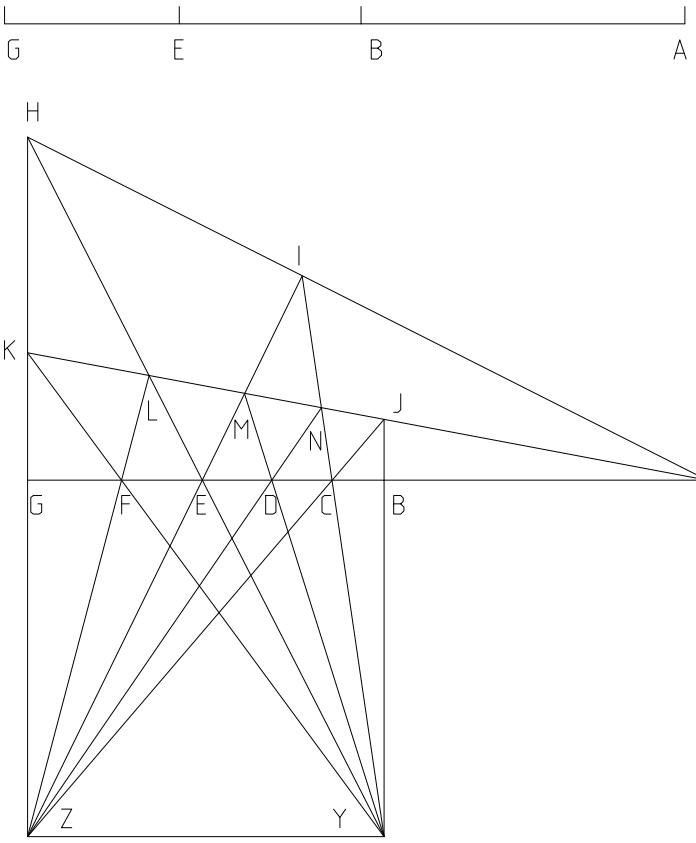
$AF := AC + CF$

$\frac{\sqrt{AF \cdot AC} - AC}{CD} = 1 \qquad \frac{\sqrt{AF \cdot AC}}{AD} = 1$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use 5^{th} root series for example.



$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

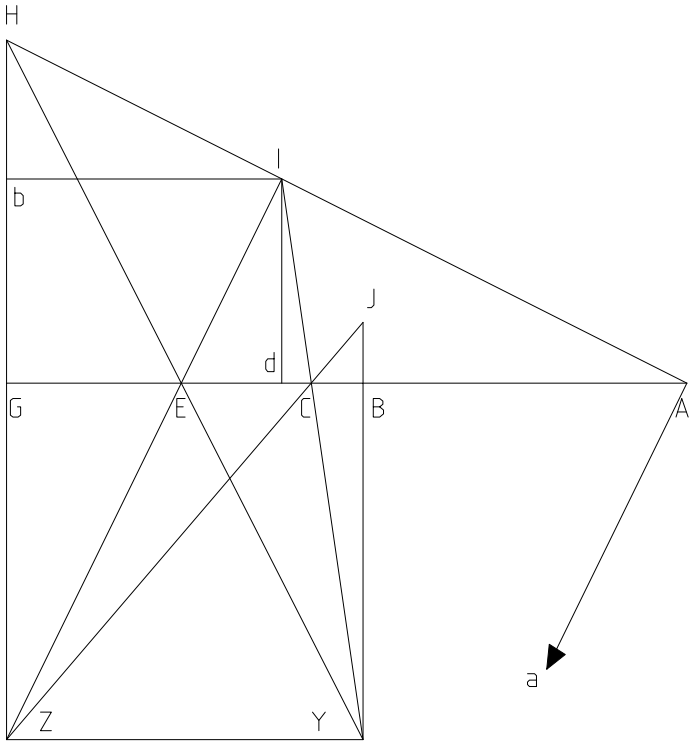
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

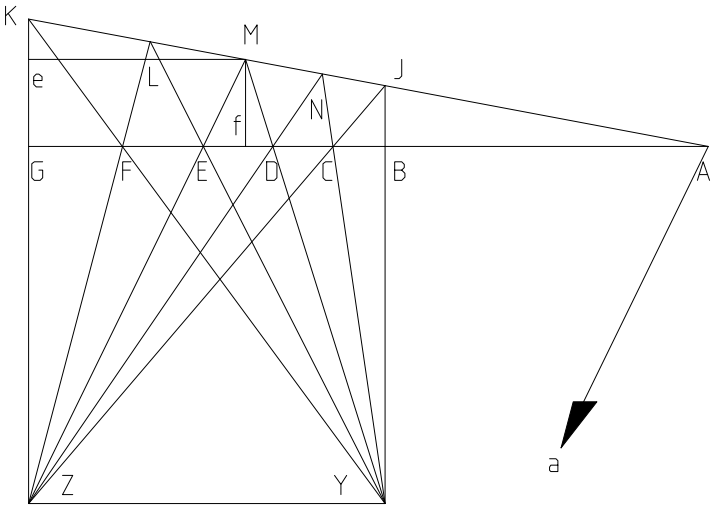
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



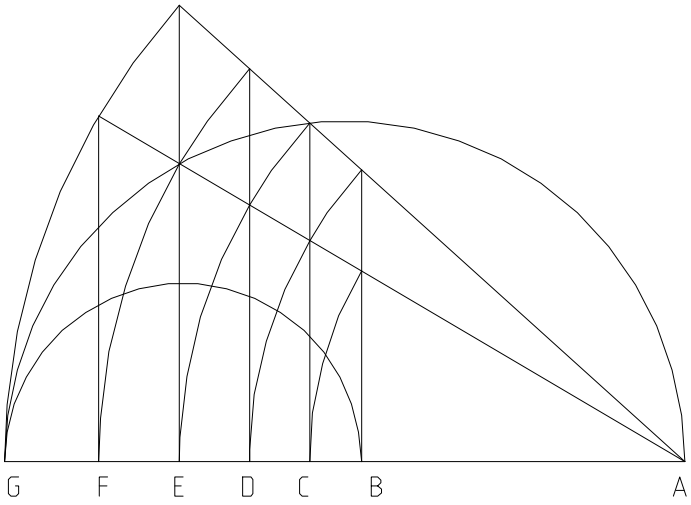


$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



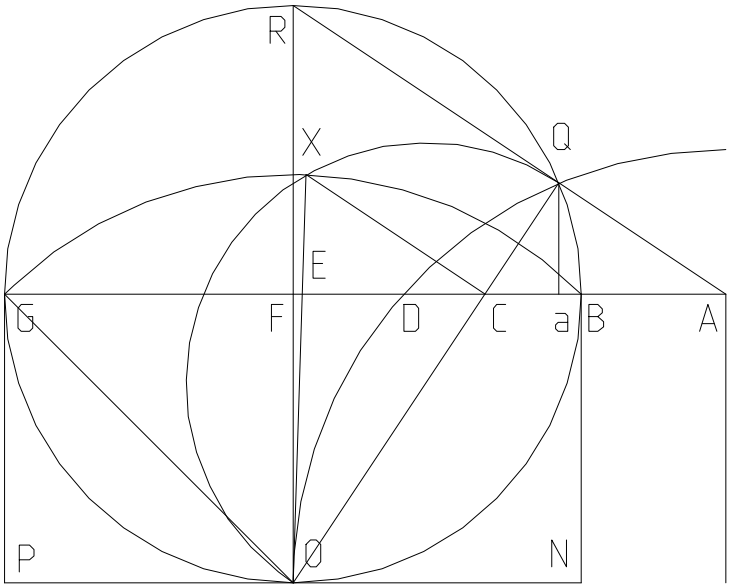
$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

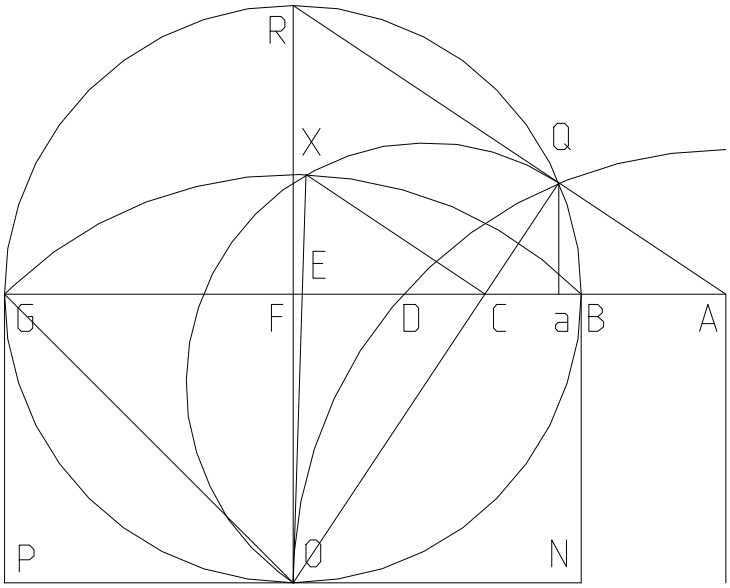
$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.





Is CX a constant?

I have had so much back work to catch up on I post dated a couple.

$AB := 54 \quad AG := 270$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FO := BF$

$AF := AB + BF \quad FR := BF$

$AR := \sqrt{AF^2 + FR^2} \quad AQ := \frac{AB \cdot AG}{AR}$

$Aa := \frac{AF \cdot AQ}{AR} \quad Qa := \frac{FR \cdot AQ}{AR}$

$Fa := AF - Aa \quad OQ := \sqrt{Fa^2 + (FO + Qa)^2}$

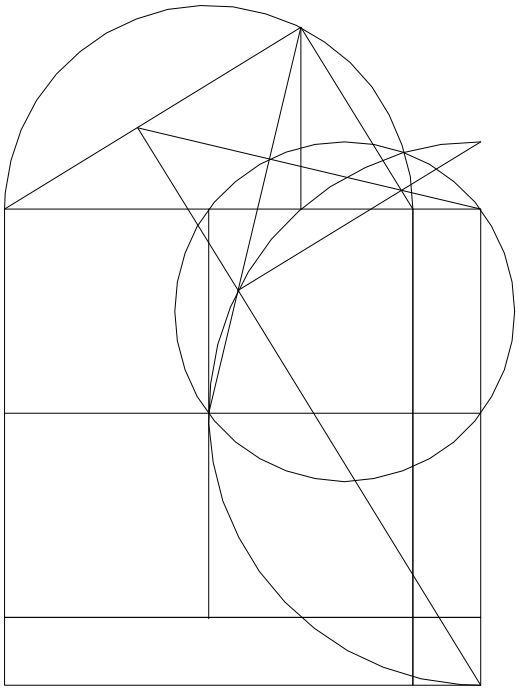
$CQ := \frac{OQ \cdot Qa}{FO + Qa} \quad OX := \sqrt{BF^2 + FO^2}$

$CO := OQ - CQ$

$\frac{\frac{OX^2}{OQ}}{CO} = 1$

Both expressions reduce to,

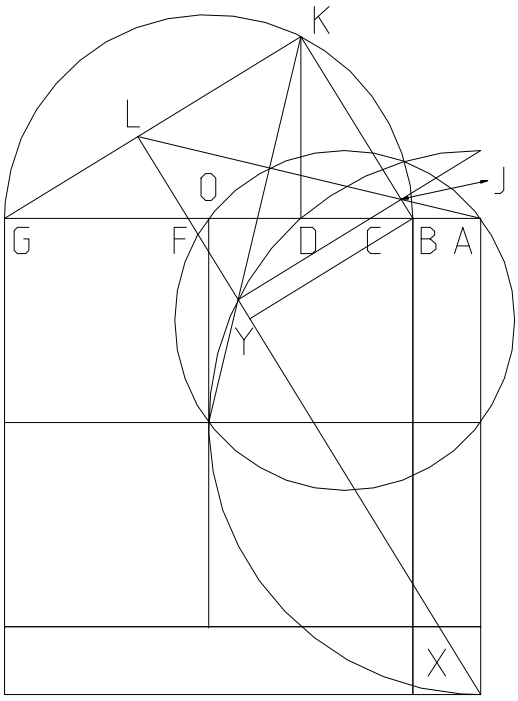
$CQ = 49.923 \qquad OQ - \frac{OX^2}{OQ} = 49.923 \qquad \frac{AG - AB}{AG + AB} \cdot \frac{\sqrt{2 \cdot (AG \cdot AB)}}{\sqrt{AB^2 + AG^2}} = 49.923$



Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.



$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

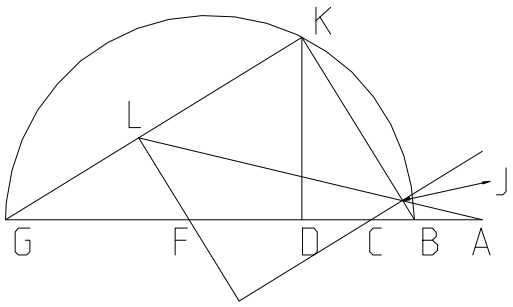
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



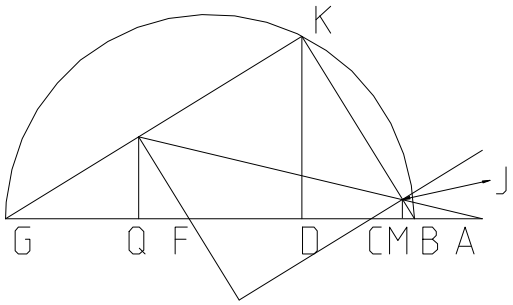
Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \quad \frac{GK}{GL} = 2.415 \quad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \quad \frac{BK}{BJ} = 8.075$$



$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

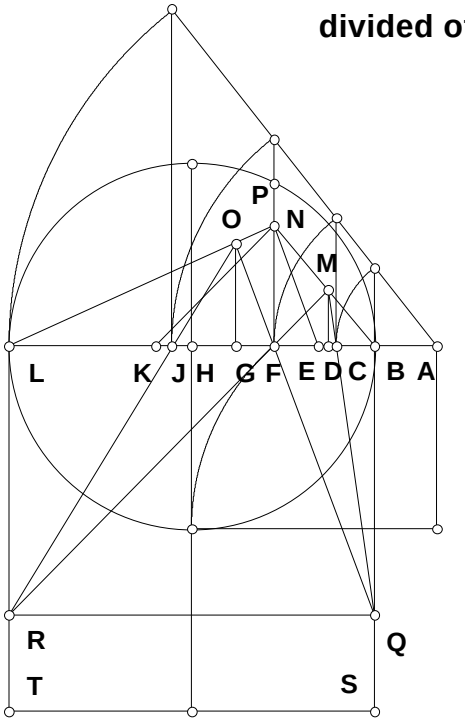
On the left is the first and last of the series, on the right is the entire series.

9.769
14.608
21.844
32.665

$\frac{AG}{BM} = 32.665$

12/20/96 Alternate Method Quad Roots

If FN:FP as BQ:BS then quad roots series can be divided off as shown in the figure.



$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \qquad BS := BL \qquad LT := BL$

$BH := \frac{BL}{2} \qquad HL := BH \qquad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \qquad FL := AL - AF \qquad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \qquad FN := \frac{BQ \cdot FP}{BS} \qquad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \qquad FG := \frac{EF \cdot FL}{EL} \qquad GO := \frac{FN \cdot FG}{EF}$

$GL := FL - FG \qquad LR := BQ \qquad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$